NON TECHNICAL SUMMARY

Including Trophic Interactions in Fish Stock Assessments in the Aleutian Islands

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OBJECTIVES:

1. Develop an assessment framework in which population dynamics models that include predator-prey interactions can be fitted formally to the data used conventionally to provide scientific management advice as well as to diet composition data.
2. Apply this framework to data for Pacific cod, Atka mackerel, and walleye pollock in the Aleutian Islands and compare the results with those from conventional single species assessments.
3. Evaluate, by means of simulation, the ability to quantify the functional form of the predator-prey relationship using the types of data and sample sizes available for the Aleutian Islands region.

NON TECHNICAL SUMMARY:

The conventional approach used to conduct stock assessments for fish species in the Aleutian Islands and Bering Sea assumes that the rate of natural mortality is independent of time (and age). The approach therefore ignores the dynamical implications of multi-species predation. This conventional approach is extended so that the dynamics of the modelled species are impacted by predation (inter- and intra-specific). The parameters of the extended population dynamics model include those determining recruitment, fishery and survey selectivity, and the feeding functional relationship. The values for, and uncertainty about, these parameters can be estimated by fitting the multi-species model to data on catches, survey and fishery catch-at-age and catch–at-length data, survey indices of abundance, and data on daily ration and the fraction of each prey species by length and by weight in the diet of each predator species.

The multi-species model is applied to data for walleye pollock, Pacific cod and Atka mackerel in the Aleutian Islands. The sensitivity of the outcomes from the assessment to the form of the feeding functional relationship is explored by considering seven alternative relationships (linear, Holling Types II and III, predator inference, predator pre-emption, Hassel-Varley, and Ecosim-like). Akaike's Information Criterion (AIC) is used to select among these relationships. The results suggest that the predator pre-emption feeding functional relationship provides the best fits to the data.

The results from the "best" multi-species model differ from those of an assessment that ignores multi-species interactions in that the estimates of age-0 abundance and total mortality for younger animals are generally much higher for assessments that allow for multi-species predation. Trends in spawning biomass are generally robust to whether or not intra- and inter-species predation is included in the assessment, but for Atka *mackerel at least, there are noteworthy differences in both trend and absolute levels of spawning biomass. Estimates of the spawning biomass and age-0 abundance are also sensitive to the choice of the feeding functional relationship, highlighting the importance of considering several alternative feeding functional relationships and developing and improving methods for selecting among them. The estimates of parameter uncertainty are sensitive to the choice of the method for representing uncertainty; in particular there is evidence that the model developed will need to be reparameterized if the Markov chain Monte Carlo (MCMC) algorithm is to be used to sample parameter vectors from the Bayesian posterior distribution for the parameters.*

The results from the fits are used as the basis for a series of Monte Carlo simulations that explore: a) the benefits of including multi-species predation in stock assessment models, b) the power to select among alternative feeding functional relationships, and c) the implications of different data collection schemes. The results suggest that the quantities on which management reference points are currently based (e.g. spawning biomass) are not greatly improved by including predation in stock assessment models. However, estimates of the quantities needed to determine the amount of food available to predators could be substantially improved by basing assessments on multi-species models. As expected, increasing diet composition sample sizes leads to less biased and more precise estimates of model outputs. However, the performance of estimation methods also depends on the uncertainty associated with other model inputs such as catch, survey indices of abundance and survey age-/length-compositions. The data are able to detect a complicated (predator pre-emption) feeding functional relationship, but the ability to correctly identify the simplest feeding functional relationship (a linear relationship) remains poor even if sample sizes are increased, perhaps due to the fact that this feeding functional relationship is nested within the other feeding functional relationships.

KEYWORDS: Atka mackerel, Aleutian Islands, Multi-species, Pacific cod, Statistical Catch-at-age Analysis, Stock Assessment, Walleye Pollock

1. BACKGROUND / INTRODUCTION / STRUCTURE OF THIS REPORT

Competition between Steller sea lions (*Eumetopias jubatis*) and fisheries for fish has been posited as a factor contributing to population declines of the western (Aleutian) stock of Steller sea lions in the late 1970s and 1980s (NRC, 2003). The three dominant groundfish fisheries in the Aleutian Islands are based on walleye pollock (*Theragra chalcogramma*), Atka mackerel (*Pleurogrammus monopterygius*), and Pacific cod (*Gadus macrocephalus*). These species are also the main prey items of Steller sea lions in the Aleutians, with overall frequencies of occurrence of 46.4%, 39.6%, and 16.1%, respectively, in scat samples (Sinclair and Zepplin, 2002). Other potential marine mammal predators of these fish include sea otters (*Enhydris lutra*), northern fur seals (*Callorhinus ursinus*), Dall's porpoise (*Phocoenoides dalli*), harbor porpoise (*Phocoena phocoena*), resident killer whales (*Orcinus orca*) and Pacific white-sided dolphins (*Lagenorhynchus obliquidens*) (Perrin *et al*., 2002).

Changes in fisheries management have been implemented to reduce the conflict between fisheries and marine mammal populations (Ferrero and Fritz, 2002). The consequences of predator-prey interactions among the key fish populations (primarily predation by Pacific cod on walleye pollock and Atka mackerel) need to be accounted for to enable reliable predictions regarding the impact of management actions on the food available for marine mammals to be made. However, at present, the assessment models for these species (e.g. Barbeaux *et al*., 2003; Lowe *et al*., 2003; Thompson and Dorn, 2003), while based on complicated age- and length-structured population dynamics models, do not include a mechanism to allow for predator-prey interactions. A variety of multi-species models, including EcoPath with Ecosim (EwE) and MSVPA have been applied to data for the Alaska region (e.g. Livingston and Methot, 1998; Trites *et al*., 1999; Livingston and Jurardo-Molina, 2000; Aydin *et al*., 2002; Ciannelli *et al*., 2004; Jurardo-Molina *et al*., 2005a, b; Guénette *et al*. 2006). However, approaches such as EwE are not designed for providing short-term tactical advice and models such as MSVPA are unable to represent uncertainty and utilize data in the same way as the stock assessments presently used for target species. There is a need therefore for a multi-species statistical catch-at-age framework that generalizes current assessment techniques by utilizing catch, survey index and survey age- and size-composition data as well as diet data for parameter estimation.

Most current multi-species models can be categorized as “whole system” models or “minimal realistic” models. “Whole system” models (such as EwE (Christensen and Walters, 2004) and Atlantis (Fulton *et al*., 2004)) tend to capture a broad range of species (or groups of species) at the cost of the ability to represent species at the level of resolution commonly incorporated in standard stock assessments models. These models often have no (e.g. Atlantis) or limited (EwE) facilities for formal parameter estimation. Rather than providing a basis for developing tactical management advice, these models provide the ability to evaluate management options in a strategic sense. In contrast, “minimal realistic” models focus on a small number of species, but model each species in more detail than is possible or common when applying “whole system” models. The number of species included in “minimal realistic models” can be chosen so that the bulk of the predation on some key species of interest is accounted for (e.g. Punt and Butterworth 1995). Approaches based on the “minimal realistic” model paradigm often also include methods for formal parameterization and hence quantification of uncertainty.

This “minimum realistic modeling” approach to including predation in assessment models has been used to model interactions among cod, capelin, polar cod, harp seals and minke whales in the Barents Sea (e.g. MULTSPEC – Bogstad *et al.*, 1997; BORMICON – Stefansson and Palsson, 1998); and among Cape hake, Cape fur seals and other predatory fish off southern Africa (Punt and Butterworth, 1995). Northern boreal shelf ecosystems are characterized by a relatively few species with strong interactions that dominate the rest of the system (Livingston and Tjelmeland, 2000) making them ideal for modelling using the “minimal realistic” modelling framework. For example, Livingston and Methot (1998) incorporated predation by Pacific cod and northern fur seals on walleye pollock into a single-species assessment of pollock in the eastern Bering Sea. Their model also considered cannibalism on small pollock by larger pollock by means of a Type I feeding functional relationship. However, since the assessment was for pollock only, both predators were modeled as known “external forcing functions”. In reality, however, there is uncertainty about the numbers of predators and their feeding functional relationships.

The “minimal realistic modeling” approach has been selected for this project because:

i) this approach generalizes the current methods of stock assessment to include predator-prey interactions, thereby facilitating direct comparison of the results from this project with those from the current assessments of the three species of interest;
ii) standard methods of fitting models and quantifying uncertainty can be applied; and
iii) the analyses focus on the species for which reliable data are available.

The report is structured around the three objectives listed in Chapter 2. Chapter 3 provides the mathematical specifications for how a standard stock assessment model (in this case the package AMAK, which has been used by AFSC scientists to conduct stock assessments of Bering Sea, Gulf of Alaska and Aleutian Islands fish stocks[1]) can be extended so that account can be taken of predation mortality and so that alternative feeding functional relationships can be compared using standard model selection criteria.

Chapter 4 outlines the application of the modelling framework developed in Chapter 3 to the actual case of walleye pollock, Pacific cod and Atka mackerel in the Aleutians Islands, Alaska. The focus for the application was to obtain a baseline model that adequately captured all the various data sources yet was nevertheless reasonably parsimonious.

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[1] AMAK employs an explicit age-structured model with the standard catch equation to fit to data on catches, survey indices of relative abundance, and survey and catch age-compositions. It includes various options for specifying selectivity-at-age and the relationship between stock and recruitment.

Studies that have involved fitting multi-species population dynamics models to data for fish and marine mammal populations in the North Pacific (e.g. Livingston and Methot, 1998; Livingston and Jurando-Molina, 2000; Jurando-Molina *et al*. 2005a, b; Guénette *et al*. 2006) and other regions (Punt and Butterworth, 1995; Schweder *et al*. 1998; Bundy and Fanning, 2005; Koen-Alonso and Yodzis, 2005) tend to suggest that the use of multi-species models will lead to different conclusions than those from single-species models. However, multi-species models typically require considerably more data than single-species models, and often rely on more assumptions than their single-species counterparts.

It might be anticipated therefore that multi-species models will lead to more accurate and precise predictions when it is possible to: 1) correctly identify functional forms for key biological processes and 2) estimate the values for the parameters for these processes adequately. In contrast, it may the case that simpler models provide more robust predictions than more complicated models (e.g. Ludwig and Walters, 1985). In the context of multi-species models, it may be that models that ignore inter-specific predation perform better at predicting many of the quantities of interest to management. Chapter 5 of this report therefore conducts a simulation evaluation of the approach developed in Chapter 3 to assess the extent to which it is possible to select among alternative feeding functional relationships and to provide more accurate and precise estimates of quantities of management interest given the uncertainty associated with the data for the example species. It also examines the value of different data collection strategies.

Chapter 6 of this report outlines some additional research priorities that arise from the results of the analyses.

2. OBJECTIVES

The objectives for the study were:

1) Develop an assessment framework in which population dynamics models that include predator-prey interactions can be fitted formally to the data used conventionally to provide scientific management advice as well as to diet composition data.
2) Apply this framework to data for Pacific cod, Atka mackerel, and walleye pollock in the Aleutian Islands and compare the results with those from conventional single species assessments.
3) Evaluate, by means of simulation, the ability to quantify the functional form of the predator-prey relationship using the types of data and sample sizes available for the Aleutian Islands region.

3. MLMAK – A MULTI-SPECIES CATCH-AT-AGE ANALYSIS

3.1 Basic framework

The steps needed to formally include predator-prey interactions into a conventional statistical catch-at-age stock assessment are:

1. The rate of natural mortality, M, needs to be made to depend on the abundance of predators and prey through a feeding functional relationship. Mortality from predators included in the model needs to separated from that due to factors not included in the model (other predators, disease, etc.).
2. The likelihood function needs to include terms based on the diet composition data. For the purposes of this project, the diet data are used to determine three types of information about predation: a) the daily ration for each predator age-class, b) the proportion of the total mass that each prey species (and "other prey") constitutes of the total mass consumed by each length-class of predator species, and c) the length-structure of the prey consumed by predators of each predator species and length-class.
3. The parameterization of the model, including the feeding functional relationship, needs to be such that current single-species assessments (AMAK in this case) are a special case of the model to enable statistically-based comparisons to be made between single-species and multi-species assessments and among alternative feeding functional relationships.

The following sections of this chapter outline the mathematical structure of the population dynamics model (Section 3.2), how the values for the parameters of the model are determined (Section 3.3), how it is possible to select among alternative functional forms for predation mortality (Section 3.4), and how uncertainty can be quantified (Section 3.5). This chapter avoids references to the specific details of the application to walleye pollock, Pacific cod and Atka mackerel in the Aleutian Islands – these details are provided in Chapter 4.

3.2 Population dynamics model

3.2.1 Basic dynamics

The numbers-at-age for the years $y_\mathrm{P}+1, y_\mathrm{P}+2, .., y_\mathrm{L}$, where y_P and y_L are respectively the first and last years considered in the model, are given by:

$$N_{k,a,y} = \begin{cases} R_{0,k} e^{\varepsilon^\mathrm{R}_{k,y}} & \text{if } a = 0 \\ N_{k,a-1,y-1} L_{k,a-1,y-1} & \text{if } 1 \le a < A_{\mathrm{L},k} \\ N_{k,A_{\mathrm{L},k}-1,y-1} L_{k,A_{\mathrm{L},k}-1,y-1} + N_{k,A_{\mathrm{L},k},y-1} L_{k,A_{\mathrm{L},k},y-1} & \text{if } a = A_{\mathrm{L},k} \end{cases} \tag{3.1}$$

where $N_{k,a,y}$ is the number of animals of age a and species k at the start of year y,

$R_{0,k}$ is the mean unfished recruitment for species k,

$L_{k,a,y}$ is the survival rate for animals of species k and age a during year y:

$$L_{k,a,y} = e^{-Z_{k,a,y}} \tag{3.2}$$

$Z_{k,a,y}$ is the total mortality rate for animals of species *k* and age *a* during year *y*,
$\varepsilon^{\mathrm{R}}_{k,y}$ is the logarithm of the deviation from the mean recruitment for species *k* in year *y*, and
$A_{\mathrm{L},k}$ is the oldest age considered for species *k*.

3.2.2 Total mortality

The total mortality on animals of age *a* and species *k* during year *y* is due to: a) fishing, b) predation by predators included in the model, and c) natural mortality due to factors not included in the model, i.e.:

$$Z_{k,a,y} = \sum_f F^f_{k,a,y} + P_{k,a,y} + M_k \tag{3.3}$$

where $P_{k,a,y}$ is the mortality rate for prey of species *k* and age *a* during year *y* due to the modeled predators,
$F^f_{k,a,y}$ is fishing mortality rate by fleet *f* on animals of species *k* and age *a* during year *y*, i.e.:

$$F^f_{k,a,y} = \begin{cases} S^f_{k,a,y}\, e^{\mu^f_k + \varepsilon^f_{k,y}} & \text{if } C^f_{k,y} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.4}$$

$C^f_{k,y}$ is the observed catch (in weight) of species *k* by fleet *f* during year *y*,
μ^f_k is the logarithm of the mean fishing mortality rate by fleet *f* on animals of species *k* over all years,
$\varepsilon^f_{k,y}$ is the logarithm of the deviation from the mean fishing mortality for species *k* and fleet *f* during year *y*,
$S^f_{k,a,y}$ is the selectivity of fleet *f* on animals of species *k* and age *a* during year *y* (see Section 4.2.3), and
M_k is the mortality rate for prey species *k* due to causes not included in the model.

The mortality rate due to modeled predation during year *y* for prey of species *k* and age *a*, $P_{k,a,y}$, is:

$$P_{k,a,y} = \sum_{r,u} V^{r,u}_{k,a,y} \tag{3.5}$$

where $V_{k,a,y}^{r,u}$ is the mortality rate during year *y* for prey of species *k* and age *a* due to predators of species *r* and age *u*:

$$V_{k,a,y}^{r,u} = N_{r,u,y}\,\phi_{k,a,y}^{r,u}\,\gamma_{k,a}^{r,u} \tag{3.6}$$

$\phi_{k,a,y}^{r,u}$ is the predation rate due to an individual predator of species *r* and age *u* during year *y* on “fully selected” prey of species *k* and age *a* (see Section 4.2.1), and

$\gamma_{k,a}^{r,u}$ is the selectivity of a predator of species *r* and age *u* for prey of species *k* and age *a* (see Section 4.2.2).

3.2.3 Numbers of prey eaten, prey mass, and daily ration

The total number of prey of species *k* and age *a* eaten during year *y* by predators of species *r* and age *u*, $E_{k,a,y}^{r,u}$, is given by:

$$E_{k,a,y}^{r,u} = \frac{V_{k,a,y}^{r,u}}{Z_{k,a,y}} N_{k,a,y}\left(1-e^{-Z_{k,a,y}}\right) \tag{3.7}$$

The number of prey of species *k* and age *a* eaten by predators of species *r* in length-class *l* during year *y* is then:

$$E_{k,a,y}^{r,l} = \sum_u \left(E_{k,a,y}^{r,u}\chi^{r,u,l}\right) \tag{3.8}$$

where $\chi^{r,u,l}$ is the age to length transition matrix for species *r* (the proportion of animals of species *r* and age *u* that are in length-class *l*).

The mass of prey species *k* (including "other") consumed during year *y* by predators of species *r* in length-class *l* , $Q_{k,y}^{r,l}$, is given by:

$$Q_{k,y}^{r,l} = \sum_u Q_{k,y}^{r,u}\chi^{r,u,l} \tag{3.9}$$

where $Q_{k,y}^{r,u}$ is the mass of prey species *k* consumed during year *y* by predators of species *r* and age *u*:

$$Q_{k,y}^{r,u} = \begin{cases} \sum_{a=0}^{A_{\mathrm{L},k}} \left(E_{k,a,y}^{r,u}\, w_{k,a}\right) & \text{for modeled prey species } k \\ Q_o^r\left(1-e^{-O_y^{r,u}}\right) & \text{for "other" prey} \end{cases} \tag{3.10}$$

$w_{k,a}$ is the mean weight of an individual of age a and species k in the population,

$O_y^{r,u}$ is the mortality due to predators of species r and age u on "other" prey:

$$O_y^{r,u} = \phi_o^{r,u} N_{r,u,y} \tag{3.11}$$

$\phi_o^{r,u}$ is the predation rate by predators of species r and age u on "other" prey (assumed to be independent of year), and

Q_0^r is the mass of "other" prey.

The model-estimate of the daily ration of a predator of species r and age u during year y is given by:

$$\hat{\Omega}_y^{r,u} = \tfrac{1}{365}\left(\sum_k Q_{k,y}^{r,u}\right) / \bar{N}_{r,u,y} \tag{3.12}$$

where $\bar{N}_{r,u,y}$ is the average number of predators of species r and age u during year y, approximated by:

$$\bar{N}_{r,u,y} = N_{r,u,y} e^{-Z_{r,u,y}/2} \tag{3.13}$$

3.2.4 Initial conditions

Each species is assumed to be in deterministic unfished equilibrium at the start of the first year considered in the model (year y_{P}). Year y_{P} is chosen to be one year before the first year for which recruitment is estimated, i.e. $y_{\mathrm{R},k} = y_{\mathrm{C},k} - A_{\mathrm{L},k} + 1$ where $y_{\mathrm{C},k}$ is the first year for which catch data are available for species k.

3.3 The objective function

The objective function consists of contributions from the data and from penalties. The model is fitted to fisheries catches, age-compositions from fisheries, research survey indices, age-compositions from surveys, and diet data from stomach samples. Three sources of diet data are used to estimate the parameters of the predation model: a) estimates of daily ration (total mass) required by each predator species at age, b) data on the relative proportions of *mass* of various prey (including "other food") in the diets of predator species at length, and c) data on the relative proportions of *lengths* of various prey species in the diets of predator species at length.

The contribution of each of the data sources to the likelihood function is outlined below.

3.3.1 Data components in the likelihood function

3.3.1.1 Catches

The contribution of the catch data to the objective function is given by:

$$L_1 = \sum_k \left(\frac{1}{(2\sigma_{\mathrm{C},k})^2} \sum_{f=1}^{n_k^f} \sum_{y=y_{\mathrm{C},k}}^{y_L} \ln\left(C_{k,y}^f / \hat{C}_{k,y}^f \right)^2 \right) \tag{3.14}$$

where $\sigma_{\mathrm{C},k}$ is the (pre-specified) coefficient of variation of the catch data for species *k*,

$\hat{C}_{k,y}^f$ is the model-estimate of the catch (in mass) of species *k* by fleet *f* during year *y*:

$$\hat{C}_{k,y}^f = \sum_{a=0}^{A_{\mathrm{L},k}} W_{k,a,y}^f N_{k,a,y} \frac{F_{k,a,y}^f}{Z_{k,a,y}} \left(1 - e^{-Z_{k,a,y}}\right) \tag{3.15}$$

$W_{k,a,y}^f$ is the mean weight in fishery *f* of an animal of age *a* and species *k* during year *y*, and

n_k^f is the number of fisheries for species *k*.

3.3.1.2 Indices of abundance

The contribution of the survey indices of abundance to the objective function is given by:

$$L_2 = \sum_k \sum_{s=1}^{n_k^s} \sum_y \frac{1}{2(\sigma_{k,y}^s)^2} \left(I_{k,y}^s - \hat{I}_{k,y}^s \right)^2 \tag{3.16}$$

where $I_{k,y}^s$ is the observed survey index of abundance for survey *s*, species *k*, and year *y*,

$\sigma_{k,y}^s$ is the standard deviation of $I_{k,y}^s$,

$\hat{I}_{k,y}^s$ is the model-estimate of the survey index of abundance for survey *s*, species *k*, and year *y*:

$$\hat{I}_{k,y}^s = q_k^s \sum_{a=0}^{A_{\mathrm{L},k}} S_{k,a,y}^s W_{k,a,y}^s N_{k,a,y} \tag{3.17}$$

$W_{k,a,y}^s$ is the mean weight in survey *s* of an animal of age *a* and species *k* during year *y*,

$S_{k,a,y}^s$ is the selectivity of survey *s* on animals of species *k* and age *a* during year *y* (see Section 4.2.3),

q_k^s is the catchability coefficient for survey *s* and species *k*, and

n_k^s is the number of surveys for species *k*.

3.3.1.3 Fishery / survey catch-at-age data

The contribution of the fishery age-composition data to the objective function is based on the assumption that the catch-at-age data are sampled multinomially from the catch, i.e.:

$$L_3 = \sum_k \sum_f \sum_{y=y_{C,k}}^{y_L} n^f_{k,y} \sum_{a=0}^{A_{L,k}} \left(A^f_{k,a,y} + 0.001 \right) \ln\left(\hat{A}^f_{k,a,y} + 0.001 \right) \qquad (3.18)$$

where $n^f_{k,y}$ is the effective sample size for species k, fleet f, and year y,

$A^f_{k,a,y}$ is the observed proportion that animals of age a constitute of the catch of species k by fleet f during year y, and

$\hat{A}^f_{k,a,y}$ is the model-estimate of the proportion that animals of age a constitute of the catch of species k by fleet f during year y, i.e.:

$$\hat{A}^f_{k,a,y} = \frac{N_{k,a,y} S^f_{k,a,y}}{\sum_{h=0}^{A_{L,k}} N_{k,h,y} S^f_{k,h,y}} \qquad (3.19)$$

The contribution of the survey catch-at-age data to the objective function follows Equation 3.18, except that the model-predictions are based on the survey selectivity pattern rather than the fishery selectivity pattern.

3.3.1.4 Fishery / survey catch-at-length data

The contribution of the fishery length-composition data to the objective function is based on the assumption that the catch-at-length data are sampled multinomially from the catch, i.e.:

$$L_4 = \sum_k \sum_f \sum_{y=y_{C,k}}^{y_L} n^f_{k,y} \sum_l \left(A^f_{k,l,y} + 0.001 \right) \ln\left(\hat{A}^f_{k,l,y} + 0.001 \right) \qquad (3.20)$$

where $n^f_{k,y}$ is the effective sample size for species k, fleet f, and year y,

$A^f_{k,l,y}$ is the observed proportion that animals in length-class l constitute of the catch of species k by fleet f during year y, and

$\hat{A}^f_{k,l,y}$ is the model-estimate of the proportion that animals in length-class l constitute of the catch of species k by fleet f during year y, i.e.:

$$\hat{A}^f_{k,l,y} = \sum_a \left(\hat{A}^f_{k,a,y} \chi^{k,a,l} \right) \qquad (3.21)$$

The contribution of the survey catch-at-length data to the objective function follows Equation 3.20, except that the model-predictions are based on the survey selectivity pattern rather than the fishery selectivity pattern.

3.3.1.5 Predator daily ration

The estimates of daily ration are assumed to be log-normally distributed, i.e. ignoring constants independent of the model parameters, the contribution of the estimate of the daily ration for a predator of species *r* and age *u* to the objective function is given by:

$$L_5 = \frac{(\ln \Omega^{r,u} - \ln \bar{\hat{\Omega}}^{r,u})^2}{2\sigma_d^2} \tag{3.22}$$

where $\bar{\hat{\Omega}}^{r,u}$ is the average over the years included in the model of $\hat{\Omega}_y^{r,u}$,

$\Omega^{r,u}$ is the observed daily ration (in kgs) of prey biomass needed to support a predator of species *r* and age *u*, and

σ_d is the (assumed) variance of the logarithm of the daily ration.

3.3.1.6 Predator prey mass-composition

The data on the proportion by length-class of various prey species, include “other food” in the diet of each predator species is assumed to be multinomially distributed, i.e. ignoring constants independent of the model parameters, the contribution of the mass-specific breakdown of the diet to the objective function is given by:

$$L_2 = \sum_r \sum_l \sum_y g_y^{r,l} \sum_k \varpi_{k,y}^{r,l} \ln \tilde{Q}_{k,y}^{r,l} \tag{3.23}$$

where $\tilde{Q}_{k,y}^{r,l}$ is the model-estimate of the fraction of the diet (in mass) during year *y* of a predator of species *r* in length-class *l* that consists of prey species *k*:

$$\tilde{Q}_{k,y}^{r,l} = Q_{k,y}^{r,l} \Big/ \sum_{k'} Q_{k',y}^{r,l} \tag{3.24}$$

$g_y^{r,l}$ is the effective sample size for the diet data on prey mass for predator species *r* and length-class *l* during year *y*, and

$\varpi_{k,y}^{r,l}$ is the observed proportion which prey species *k* constitutes of the diet during year *y* of predators of species *r* in length-class *l*.

3.3.1.7 Predation prey length-composition

The data on the length-composition of the diet of various prey species in the stomachs of each length-class of predator is assumed to be multinomially distributed, i.e. ignoring constants independent of the model parameters, the contribution of the length breakdown of each prey-species in the stomachs of each predator species to the negative of the logarithm of the likelihood function is given by:

$$L_3 = \sum_r \sum_l \sum_k H_k^{r,l} \sum_m \tau_{k,m}^{r,l} \ln \tilde{T}_{k,m}^{r,l} \tag{3.25}$$

where $H_k^{r,l}$ is the effective sample size for the diet data on lengths of prey of species k for predator species r and length-class l ,

$\tau_{k,m}^{r,l}$ is the observed fraction which animals in length-class m make up of the component of the diet of predators of species r in length-class l that is made up prey of species k,

$\tilde{T}_{k,m}^{r,l}$ is the model-estimate of the fraction which animals in length-class m make up of the component of the diet of predators of species r in length-class l that is made up prey of species k:

$$\tilde{T}_{k,m}^{r,l} = \frac{\sum_y n_y^{r,l} \sum_a \left(E_{k,a,y}^{r,l} \, \chi^{k,a,m} \right)}{\sum_y n_y^{r,l} \sum_{a'} E_{k',a',y}^{r,l}} \tag{3.26}$$

$n_y^{r,l}$ is the number of stomachs from predators of species r in length-class l that were sampled during year y.

3.3.2 Model Constraints

In addition to the contributions to the objective function based on data, the values for the parameters of the model are constrained based on penalty functions.

3.3.2.1 Recruitment

Two penalty functions pertain to recruitment. The first penalty function is based on the assumption that the recruitment deviations for the years prior to a pre-specified year $y_{\mathrm{E},k}$ are normally distributed about a mean of 0, i.e.:

$$\Lambda_2 = \sum_k \left(\frac{1}{2(\sigma_{\mathrm{R},k})^2} \sum_{y=y_{\mathrm{R},k}}^{y_{\mathrm{E},k}} \left(\varepsilon_{k,y}^{\mathrm{R}} \right)^2 + \left(y_{\mathrm{E},k} - y_{\mathrm{R},k} + 1 \right) \ln\left(\sigma_{\mathrm{R},k} \right) \right) \tag{3.27}$$

where $\sigma_{\mathrm{R},k}$ is the (assumed) standard deviation of the fluctuations in recruitment.

The second recruitment penalty is based on a stock-recruitment relationship, i.e.:

$$\Lambda_3 = \sum_k \left(\frac{1}{2\left(\sigma_{\mathrm{R},k}\right)^2} \sum_{y=y_{\mathrm{E},k}}^{y_{\mathrm{L}}} \left(\ln N_{k,y,0} - \ln \hat{R}_{k,y} + \left(\sigma_{\mathrm{R},k}\right)^2 / 2 \right)^2 + (y_{\mathrm{L}} - y_{\mathrm{E},k} + 1) \ln\left(\sigma_{\mathrm{R},k}\right) \right) \tag{3.28}$$

where $\hat{R}_{k,y}$ is the model-estimate of the number of age-0 animals of species k at the start of year y, i.e.:

$$\hat{R}_{k,y} = \frac{4 h_k R_{0,k} \, B_{k,y}}{(1-h_k) B_{0,k} + (5h_k - 1) B_{k,y}} \tag{3.29}$$

$B_{k,y}$ is the spawning biomass of species *k* during year *y*:

$$B_{k,y}=\left(\sum_{a=1}^{A_{\mathrm{L},k}}\omega_{k,a}\,m_{k,a}\,N_{k,a,y}\right)(L_{k,a,y})^{\rho_k} \tag{3.30}$$

h_k is the steepness of the stock-recruitment for the species *k* (the expected recruitment when the spawning biomass of species *k* is reduced to 20% of that in an unfished state, $B_{0,k}$),

$$B_{0,k}=\left(\sum_{a=1}^{A_{\mathrm{L},k}}\omega_{k,a}m_{k,a}N_{k,a,y_{\mathrm{P}}}\right)(L_{k,a,y_{\mathrm{P}}})^{\rho_k} \tag{3.31}$$

$\omega_{k,a}$ is the weight, at the time of spawning, of an animal of age *a* and species *k*,

$m_{k,a}$ is the proportion of animals of species *k* and age *a* that are mature, and

ρ^k is the fraction of the year in which spawning occurs for species *k*.

3.3.2.2 Fishery / Survey Selectivity

If selectivities are modeled using a separate selectivity coefficient for each age and year (as opposed to being calculated from a logistic curve), a penalty is placed on the second differences of the deviations from one age to the next in a given year:

$$\Lambda_4=\sum_k\sum_f\sum_y\sum_{a=0}^{A_{\mathrm{L},k}-2}\lambda_{1,k}\left(\eta^f_{k,y,a+2}+\eta^f_{k,y,a}-2\eta^f_{k,y,a+1}\right) \tag{3.32}$$

where $\lambda_{1,k}$ is the weight assigned to this penalty for species *k*, and

$\eta^f_{k,y,a}$ is the logarithm of the selectivity for animals of species *k* and age *a* during year *y* by fleet *f*.

A penalty is also placed on the average selectivity deviation over all years and ages for each combination of fleet and species, to force selectivity to tend to be flat, i.e.:

$$\Lambda_5=\sum_k 20\sum_f\left(\bar{\eta}^f_k\right)^2 \tag{3.33}$$

where $\bar{\eta}^f_k$ is the mean selectivity deviation for fleet *f* and species *k* over all years and ages.

Penalties analogous to Equations 3.32 and 3.33 are imposed on the survey selectivities.

3.3.2.3 Fishing mortality

A weak penalty is placed on fishing mortality during the early phases of the estimation process. This penalty constrains fishing mortality for each year and species to be 0.2yr^{-1}., i.e.:

$$\Lambda_6 = 0.001\sum_k \sum_{y=y_{C,k}}^{y_L} \left(\sum_f e^{\mu_k^f + \varepsilon_{k,y}^f} - 0.2 \right)^2 \tag{3.34}$$

A penalty is also placed on the average fishing mortality deviation over all years, i.e.:

$$\Lambda_7 = \sum_k 20 \sum_f \left(\bar{\varepsilon}_k^f \right)^2 \tag{3.35}$$

where $\bar{\varepsilon}_k^f$ is the average fishing mortality deviation over all years for fleet f and species k.

3.4 Model selection

A variety of approaches have been developed historically to model predator-prey interactions (May *et al.*, 1979; Berryman, 1992; Skalski and Gilliam, 2001) based on how a predator's feeding rate is assumed to respond to changes in prey and predator densities (i.e. the feeding functional relationship). A variety of alternative forms of predator functional response, from simple linear to more complicated representations, have been proposed and compared using data from fisheries and similar sources (e.g. Koen-Alonso and Yodzis, 2005).

AIC (Burnham and Anderson 2000), a likelihood-based method for selecting among models, is used to select among alternative predator-prey feeding functional relationships for a given combination of assessment and predator-prey data for the three stocks. Alternative methods (such as DIC, Spiegelhalter *et al.* 2002, and the Bayes Factor, Kass and Raftery, 1995) were examined as potential ways select among alternative models, but the values for the quantities needed to use these methods proved to be too unstable.

3.5 Quantifying uncertainty

Comparisons are made between outputs from models that are based on different data sources (e.g. models with and without diet data). AIC and similar methods cannot be used for this purpose because these methods require the models to be based on the same data set. Instead of comparing models based on different data sets using model selection methods, the point estimates of various derived outputs (e.g. estimates of annual recruitment and spawner biomass) and their estimated uncertainty are compared.

Ideally, the uncertainty associated with the model outputs from different model formulations could be compared using posterior distributions obtained from the Markov chain Monte Carlo (MCMC) algorithm (Hastings, 1970; Gelman *et al.*, 1995). This algorithm explores the posterior distribution by starting from a pre-specified vector of parameter values (with its associated posterior density). It uses a "jump function" to select a new set of candidate values for the parameters and, based on the ratio of the posterior density at the new point to that for the current point, decides whether to move to ("accept") the new point (the probability of moving is defined by the ratio of the two posterior densities) or to reject it. If accepted, the algorithm resumes from the new point, otherwise, it resumes from the current point. The posterior distribution (specifically the uncertainty associated with various model outputs) is quantified by summarizing the set of points that are accepted. Although conceptually straightforward, application of the MCMC algorithm for a real problem requires careful consideration of model parameterization and whether the algorithm has been run for sufficiently long that it has converged adequately, so that the accepted points are a random sample from the posterior distribution. Whether convergence has occurred is examined by applying the diagnostic statistics developed by Geweke (1992), Heidelberger and Welch (1983), and Raftery and Lewis (1992) and by examining the extent of auto-correlation among the samples in the chain.

Although it was possible to determine posterior distributions for the model outputs for some of the model configurations using the MCMC algorithm, this was computationally infeasible in general. Rather than relying on MCMC for all cases, the uncertainty associated with the model outputs was also determined using asymptotic methods and (parametric) bootstrapping.

4. APPLICATION TO ALEUTIAN ISLANDS POLLOCK, COD, AND ATKA MACKEREL

The conventional catch-at-age-based stock assessments for walleye pollock, Pacific cod and Atka mackerel (e.g. Barbeaux *et al*., 2003; Lowe *et al*., 2003; Thompson and Dorn, 2003) are age-structured (although selectivity for Pacific cod is length-based). Recent assessments of pollock and mackerel have been conducted using AMAK while cod assessments (for the entire Bering Sea and Aleutian Islands region) have been based on the assessment package Synthesis. These assessments include specifications for the population dynamics model and the observation model, but contain no predator-prey interactions and do not make use of information on diets based on stomach samples.

This Chapter contrasts assessments of the three species based on a single-species stock assessment method with those based on MLMAK. Sections 4.1 and 4.2 outline the data on which the analyses are based and the additional specifications needed to implement MLMAK for Pacific cod, Atka mackerel and walleye pollock in the Aleutian Islands. Section 4.3 outlines how the various data sources are weighted and how the parameters of the model in Chapter 3 are estimated. Section 4.4.1 contrasts the seven alternative feeding functional relationships and uses AIC to identify a best model, Section 4.4.2 assesses the extent to which the “best” model can mimic the available data, and Section 4.4.3 compares the outputs from the selected multi-species model with the single-species assessments. Section 4.4.4 evaluates the precision of the estimates of two key management-related quantities using the three methods of determining parameter uncertainty outlined in Section 3.5.

4.1 Data used

MLMAK uses two sets of data types: one set that is assumed to be subject to measurement error and a second set that is considered to be known exactly. Appendix A outlines the data on which the analyses of this Chapter are based and the sources for those data. The bulk of the data are those on which the single-species assessments for walleye pollock, Pacific cod and Atka mackerel have been based. However, the data also include information on the breakdown of the diet for each modelled species in mass by prey species, the length-frequency of each of the modelled species in the diets of each modelled predator species, and the daily ration for each predator age-class.

The analyses are based on there being one fishery for walleye pollock and Atka mackerel and three fisheries (trawl, longline and pot) for Pacific cod. The assessments are based on a single survey for each species (which provides age-composition data for pollock and mackerel, and length-composition data for cod).

4.2 Additional model specifications

4.2.1 Predator functional response

Seven alternative forms ("Types") of predator functional response (the predation rate by an individual predator at a given density of prey and predators – see Eqn 3.6) are considered. The first (Type I) is a simple linear response:

$$\phi_{k,a,y}^{r,u} = \theta^{r,u} v_k^r \tag{4.1}$$

Where v_k^r is predation mortality due predators of species *r* on prey of species *k*, and

$\theta^{r,u}$ determines the extent to which predation mortality changes with age (in the absence of fluctuations in predator and prey numbers for those feeding functional relationships that depend on predator and prey numbers):

$$\theta^{r,u} = 1 + \omega^r \tilde{\omega}^r / (u + \tilde{\omega}^r) \tag{4.2}$$

ω^r is fraction by which predation mortality is higher at age 0 than at infinite age, and

$\tilde{\omega}^r$ determines the rate at which predation mortality drops with age.

The six alternative predator functional responses (Skalski and Gilliam, 2001; Koen-Alonso and Yodzis, 2005) are[2]:

Holling Type II:

$$\phi_{k,a,y}^{r,u} = \frac{v_k^r \theta^{r,u} \left[1 + \tilde{v}_k^r\right]}{1 + \tilde{v}_k^r \, \Phi_y^{r,u}} \tag{4.3a}$$

where $\Phi_y^{r,u}$ is the number of prey available to predators of species *r* and age *u* during year *y* relative to the number in year y_P:

$$\Phi_y^{r,u} = \frac{\sum\limits_{k,a} \left(N_{k,a,y} \gamma_{k,a}^{r,u} \right)}{\sum\limits_{k,a} \left(N_{k,a,y_p} \gamma_{k,a}^{r,u} \right)} \tag{4.4a}$$

Equation 4.3a is parameterized so that $v_k^r \, \theta^{r,u}$ is the predation mortality due to predators of species *r* and age *u* on "fully-selected" prey of species *k* in an unfished state (i.e. $\Phi_y^{r,u} = 1$).

Holling Type III:

$$\phi_{k,a,y}^{r,u} = \frac{v_k^r \theta^{r,u} \left(1 + \tilde{v}_k^r\right) \left(\Phi_y^{r,u}\right)^{\delta_k^r - 1}}{1 + \tilde{v}_k^r \left(\Phi_y^{r,u}\right)^{\delta_k^r}} \tag{4.3b}$$

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[2] For ease of presentation, the same symbols (v_k^r, $\tilde{v}_k^r$, $\tilde{\tilde{v}}_k^r$ and δ_k^r) are used for the parameters of these functional responses, even though these parameters do not have the same meaning for each form.

Predator interference:

$$\phi^{r,u}_{k,a,y} = \frac{v^r_k \theta^{r,u} \left[1+\tilde{v}^r_k\right]}{1+\tilde{v}^r_k \Phi^{r,u}_y + \tilde{\tilde{v}}^r_k \left(\Psi^{k,a}_y - 1\right)} \tag{4.3c}$$

where $\Psi^{k,a}_y$ is a measure of the number of predators who would consume prey of species k and age a during year y relative to the number in year y_P:

$$\Psi^{k,a}_y = \frac{\sum\limits_{r,u}\left(N_{r,u,y}\gamma^{r,u}_{k,a}\right)}{\sum\limits_{r,u}\left(N_{r,u,y_p}\gamma^{r,u}_{k,a}\right)} \tag{4.4b}$$

Predator pre-emption:

$$\phi^{r,u}_{k,a,y} = \frac{v^r_k \theta^{r,u} \left[1+\tilde{v}^r_k\right]}{\left(1+\tilde{v}^r_k\,\Phi^{r,u}_y\right)\left[1+\tilde{\tilde{v}}^r_k \left(\Psi^{k,a}_y - 1\right)\right]} \tag{4.3d}$$

Hassel-Varley:

$$\phi^{r,u}_{k,a,y} = \frac{v^r_k \theta^{r,u} \left[1+\tilde{v}^r_k\right]}{\tilde{v}^r_k \Phi^{r,u}_y + \left(\Phi^{r,u}_y\right)^{\delta^r_k}} \tag{4.3e}$$

EcoSim:

$$\phi^{r,u}_{k,a,y} = \frac{v^r_k \theta^{r,u}}{1+\tilde{v}^r_k (\Psi^{k,a}_y - 1)} \tag{4.3f}$$

4.2.2 Predator-prey size selectivity

The selectivity by predators of species r and age u for prey of species k and age a is modeled using a gamma function:

$$\gamma^{r,u}_{k,a} = \left(G^{r,u}_{k,a} / \tilde{G}^r\right)^{\alpha^r - 1} \exp\left[-\left(G^{r,u}_{k,a} - \tilde{G}^r\right)/\beta^r\right] \tag{4.5}$$

where $G^{r,u}_{k,a}$ is the logarithm of the ratio of the expected length of an animal of species r and age u to that of an animal of species k and age a,

$\tilde{G}^r = \left(\alpha^r - 1\right)\beta^r$, is the value of $G^{r,u}_{k,a}$ at which predator selectivity is 1, and

α^r, β^r are the parameters of the predation selectivity function for predator species r.

4.2.3 Fishery and survey selectivity

Selectivity varies between the surveys (Eqn 3.17) and the fisheries (Eqn 3.4) for each species and can be modeled using one of two approaches: 1) an asymptotic logistic curve, or 2) age-specific coefficients. In the former case, time-varying selectivity can be modeled using logistic curves for pre-specified periods of years, while in the latter case time-varying selectivity can be modeled by estimating age-specific coefficients for periods of years. Selectivity for species *k* is assumed to be independent of age above age $A_{\mathrm{M},k}$.

Asymptotic logistic selectivity is modeled as:

$$S^{f}_{k,a,y} = \left(\frac{1}{1+e^{-\mathrm{B}^{f}_{k,y}\left(a-\mathrm{A}^{f}_{k,y}\right)}} \right) \tag{4.6}$$

where $\mathrm{A}^{f}_{k,y}, \mathrm{B}^{f}_{k,y}$ are the parameters of the selectivity pattern for fleet *f* when it fishes for species *k* during year *y*.

Age-specific coefficients for fishery selectivity are modeled as:

$$S^{f}_{k,a,y} = \begin{cases} e^{\eta^{f}_{y,a}} & \text{if } a \le A_{\mathrm{M},k} \\ e^{\eta^{f}_{y,A_{\mathrm{M},k},y}} & \text{otherwise} \end{cases} \tag{4.7}$$

An analogous set of equations to equations 4.6 and 4.7 are used to model survey selectivity.

4.3 Model parameterization

Age-specific selectivity for the pollock, mackerel and the three cod fisheries is modeled using the "selectivity coefficients" approach (i.e. estimable selectivity parameters for each age-class). Fishery selectivity is allowed to vary over time for mackerel (i.e. an estimated selectivity parameter for each age-class and year), but not for pollock or cod. Selectivity for the trawl fishery for cod is assumed to have changed in 1993 (based on changes in the length-frequency of the catch by the trawl fleet at that time) although the selectivity patterns for the longline and pot fisheries for cod are assumed to be time-invariant. Survey selectivity is modeled using the "selectivity coefficients" approach for all three species and does not vary over time. Fishery selectivity is assumed to be flat above age 12 for pollock, and age 10 for mackerel and cod (i.e. $A_{\mathrm{M},k}$ for the fisheries is 12 for pollock and 10 for cod and mackerel) while survey selectivity is assumed to be flat above age 10 for all three species.

Table 4.1 lists the parameters of the population dynamics model for the application to Pacific cod, Atka mackerel and walleye pollock. The application is based on setting the plus-group age-class, $A_{\mathrm{L},k}$, to 15yr, 15yr and 12yr respectively for pollock, mackerel and cod. The first year included in the penalty related to the stock-recruitment relationship

($y_{\mathrm{E},k}$ in Equation 3.28) is set to 1978 for pollock and cod, and 1977 for mackerel, based on the specifications in the original AMAK and Synthesis assessments.

The pre-specified parameters of the population dynamics model include the steepness of the stock-recruitment relationship (set to 0.6 for cod and pollock and 0.8 for mackerel), the survey catchability coefficient (set to 1), the natural mortality rate for cod (0.37yr^{-1}), and the logarithm of the mass of "other" prey ($\ell\mathrm{n}Q_0^r = 9.32$). The values for steepness and survey catchability are set to the values for the single-species assessments. Unlike the situation for cod and mackerel, the natural mortality rate for cod is pre-specified. This is because there are no age-composition data for cod on which estimation of this parameter could be based.

The analyses of this chapter are based on selecting the same predation function for all three species, primarily for ease of presentation.

4.3.1 Data weighting

The effective sample sizes assigned to the fishery age-compositions for pollock and mackerel (Tables A.2 and A.3) are set to the values used in the 2003 stock assessments while the effective samples size for the cod length-frequency data were chosen so that the impact of the penalties on selectivity (Equations 3.32 and 3.33) does not dominate the estimation of selectivity. Application of the approach of McAllister and Ianelli (1999) leads to values for the effective sample sizes larger than this. However, adopting such effective sample sizes could lead to over-weighting the fishery length-composition data for cod. The effective sample sizes for the survey age-composition data for pollock and mackerel are also set to the values assumed in the assessment, while the effective sample sizes for the cod survey length-frequency data are set to 200. The standard deviation for the observation error about the catch biomass data (σ_C) is set to 0.05 to ensure that the model mimics the historical removals closely.

The effective sample sizes for the diet data are set to the minimum of the number of stomachs examined and a maximum of 20. This is to prevent years in which large numbers of stomachs were examined (see, in particular, Tables A.12, A.14, A.16) from dominating the fits. The standard deviation of the logarithm of the diet data, σ_d, is set to 0.05. This value is essentially arbitrary, but was chosen to ensure that the predation model mimics the daily rations well.

The value of parameter that determines the magnitude of the penalty on the second derivative of the selectivities ($\lambda_{1,k}$; Eqn 3.32) is set to 0.2 for the fisheries for cod and pollock, and 0.4083 for the fishery for mackerel; the values for this parameter for pollock and mackerel are taken from the 2003 assessments, while $\lambda_{1,k}$ for cod is set equal to that for pollock. The value for $\lambda_{1,k}$ for the survey selectivities is set to 0.25 for all three species. Weak penalties are imposed on the parameters that determine the predation functions to ensure that they do not tend to infinity for those cases in which the data are sparse or predation interactions appear weak (e.g. mackerel consumption of cod).

4.4 Results and discussion

4.4.1 Selection of a base-case model

Table 4.2 contrasts seven predation models based on different choices for the feeding functional relationship using AIC. The “predator pre-emption” feeding functional relationship (see Eqn 4.3d) leads to the best fit to the data according to AIC; the remaining models lead to fits that differ from that for Eqn 4.3d by 300+ units of AIC, suggesting that the data strongly support Eqn 4.3d over the other feeding functional relationships. Eqn 4.3d therefore forms the base-case for the remaining calculations of this chapter. Note that the “no predation” model is not included in Table 4.2 because the likelihood function for this model is not the same as that for the “with predation” models because the “no predation” model is not fitted to the diet data.

Table 4.3 explores the fit of the base-case model compared to the other models (including the “no predation” model) in terms of the values for the various contributions to the objective function (the assorted penalties are combined into a single number in Table 4.3 for ease of presentation). The values for the contributions to the objective function are expressed as differences from those for the “predator pre-emption” model (i.e. negative values in Table 4.3 indicate fits that are better than those for the “predator pre-emption” model and *vice versa*). The base-case model leads to the best fits to the daily rations, the weight composition of the diets, the catch biomass data, and the survey indices of abundance. In contrast, other “with predation” models provide better fits to the fishery age-/length-compositions (Type II), the survey age-/length-compositions (predator interference), and the length-composition of the diets (Type III). The “no predation” model leads to fits to the catch biomass, fishery age-/length-compositions, and the survey indices of abundance that are better than those of the “predator pre-emption” model. However, this is perhaps not unexpected because the “no predation” model has almost as any many parameters as the “with predation” models, but does not have to fit nearly as many data sources.

Although AIC suggests that the “predator pre-emption” model leads to the best fits to the data (Table 4.2), it is not necessarily the case that key management-related model outputs are highly sensitive to how predation (both inter- and intra-species) is included in the assessment model. Figures 4.1 and 4.2 therefore contrast the estimated time-trajectories of spawning biomass and age-0 abundance for each of the seven “with predation” models. In general, all of the predation models lead to similar trends in spawning biomass for cod, particularly after about 1990 (Fig. 4.1) and there are few differences among the predation models in terms of the estimated time-trajectory of age-0 abundance of cod. This result is perhaps not too surprising because cod tends not be preyed on to a substantial extent by pollock and mackerel. There are more differences among the predation models in terms of the time-trajectories of spawning biomass and age-0 abundance for pollock and mackerel. For example, the Type I and Type III predation models lead to notably different trends in mackerel spawning biomass (Fig. 4.1) while the type III predation model leads to markedly more variable estimates of the abundance of age-0 pollock (Fig. 4.2). The base-case model generally leads to among the highest levels of age-0 abundance for both pollock and mackerel. In general, the differences among the predation models, in terms of trends in spawning biomass, are greatest for the early years (although this is not the case for pollock for which the results from the Type I and Type

III feeding functional relationship are quite different from the results of models based on the other feeding functional relationships even for recent years).

4.4.2 Model fit diagnostics for the base-case model

Figures 4.3-4.6 show the observed and base-case model-predicted survey indices of abundance, survey age-/length-compositions, fishery age-/length-compositions, and fishery catch biomasses respectively. Figures 4.7 and 4.8 summarize the fits to the survey and fishery age-/length-compositions using bubble plots. The model mimics the survey indices of abundance adequately, with the model estimates intersecting all but four of the 95% confidence intervals for the data (Fig. 4.3). The model follows the point estimates of abundance for mackerel and cod, but does not mimic the point estimates of abundance for pollock particularly well (although the recent estimates of pollock abundance are very imprecise). The model fits the survey age-compositions for pollock and cod well (Figs 4.4a and 4.4b). However, the survey length-composition data for cod vary substantially from one year to the next and the model is consequently not able to mimic these data well (Fig. 4.4c). In contrast to the situation for the survey length-composition data, the model fits the fishery length-compositions for cod (Figs 4.5c, 4.5d, and 4.5e) adequately. The model also fits the mackerel fishery age-compositions closely (Figure 4.5b). However, the fits to the fishery age-compositions for pollock, which also exhibit considerable inter-annual variation, are poor (Figure 4.5a). The model mimics the catches almost exactly, although the model is not able to match the catches of cod by the trawl fishery in some recent years exactly (Fig. 4.6).

Figure 4.9 shows the fit of the base-case model to the information on daily rations. The error bars in Fig. 4.9 are based on the assumed standard error ($\sigma_d = 0.05$) assigned to be logarithms of the daily rations. As expected given the value for this standard error, the model mimics the rations almost exactly. However, the ability to mimic the rations for ages 2-4 is poorer than is the case for the remaining ages. Figure 4.10 shows the fits to the length-composition of pollock and mackerel of various lengths in the diets of pollock, mackerel and cod of various lengths. Results are only shown for those combinations of prey and predator for which the sample sizes are reasonably large (see Tables A.18, A.20 and A.22). The model mimics the prey lengths of pollock in the stomachs of pollock and mackerel (Figs 4.10a and 4.10b) and of Atka mackerel in the stomachs of Pacific cod (Fig. 4.10d) well. However, the model tends to under-predict the lengths of walleye pollock in the stomachs of Pacific cod (Figure 4.10c). Figures 4.11 and 4.12 summarize the fits to the diet length- and weight-composition data in the form of bubble plots. In general, the residuals are small. However, there are some cases in which there are large residuals (e.g. the fraction of "other prey" in the diet of mackerel; Fig. 4.11, centre right panel).

4.4.3. Comparison of the "with" and "no" predation models

Figures 4.13 and 4.14 contrast the estimated time-trajectories of spawning biomass and age-0 abundance from the base-case "with predation" model and the "no predation" model. The results for Pacific cod are similar for the two models and the time-trajectory of spawning biomass for pollock is also similar until the late 1990s when the "no predation" model indicates an increase in the abundance of pollock, but the "with predation" model suggests a decline. The spawning biomass of mackerel is estimated to be lower for the "with predation" model. The estimates of age-0 abundance are higher for the "with predation" model for pollock and cod (as expected). However, this is not the case for mackerel (Fig. 4.15), although this is consistent with the differences in the estimates of spawning biomass. As expected, natural mortality for younger ages is higher for the "with predation" model (Figs 4.15 and 4.16), with the consequence that the ages for which total mortality is highest for pollock and mackerel are the younger rather than the older age-classes (Fig. 4.16). The patterns over time of total mortality for Pacific cod are robust to whether predation is explicitly modelled or not (Fig. 4.16, right panels), but this is less evident for pollock and mackerel (Fig. 4.16, left and centre panels).

Fishery and survey selectivity are generally robust to whether predation is modelled explicitly or not (Figs 4.17 and 4.18) although the survey selectivity pattern for mackerel from the "with predation" model is domed-shaped rather than being asymptotic, as is the case for the "no predation" model. The difference in survey selectivity for mackerel between the two models is consistent with the differences in absolute biomass between the two models for this species.

Figure 4.19 shows the estimated predator selectivity patterns. As expected, predation mortality is greatest on younger (smaller) animals and the average age of prey increases with predator age.

4.4.4 Parameter uncertainty

The results from the MCMC algorithm are based on 5,000,000 cycles, of which the first 1,000,000 were taken to be a 'burn', and the resulting chain was thinned by basing posterior inference on every 2,000'th element in the resulting chain. Figures 4.20 and 4.21 provide diagnostic plots for the objective function and the spawning biomass in 1994. The panels in these diagnostic plots show the trace, the posterior density function (estimated using a normal kernel density), the correlation at different lags, the 50-point moving average against cycle number (dotted line in the rightmost panels), and the running mean and running 95% probability intervals (solid lines in the rightmost panels). All plots suggest that there is still considerable autocorrelation in the chain even with a thinning ratio of 2,000. Moreover, the auto-correlation for the objective function tends to asymptote at high lags suggesting that it will be necessary to reparameterize the model if improved convergence diagnostics are to be obtained and that running the MCMC algorithm for longer is unlikely to resolve the problem[3]. Results (not shown here) indicate that the MCMC algorithm converges fairly rapidly for the "no predation" model, indicating that it is the introduction of predation or its parameterization that has led to the convergence problems evident in Figures 4.20 and 4.21.

Figures 4.22 and 4.23 contrast the coefficients of variance for age-0 abundance and spawner abundance. The estimated coefficients of variation differ quite markedly among the three methods of measuring parameter uncertainty. The upper panels compare the CVs from three methods for measuring this uncertainty: a) inverting the Hessian matrix ("asymptotic"), b) using parametric bootstrapping (500 replicates) ("bootstrap") and c) applying the Markov chain Monte Carlo algorithm to draw parameter vectors from the

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[3] 5,000,000 cycles of the MCMC algorithm required a month of run time.

multivariate posterior distribution ("Bayesian"), while the lower panels compare the asymptotic coefficients of variation for the base-case "with predation" model with those for the "no predation" model.

The "Bayesian" results should be interpreted with some caution because the results in Figs 4.20 and 4.21 indicate that the MCMC algorithm probably failed to converge. Specifically, the bootstrap indicates much poorer precision than the Bayesian and asymptotic methods for recruitment (Fig. 4.22, upper panels) while the asymptotic method generally indicates higher coefficients of variation for spawning biomass than the Bayesian and bootstrap methods (Fig. 4.23, upper panels). In general, the estimates of recruitment for the earliest and most recent years are the least precise according to the Bayesian and asymptotic methods and this also evident to some extent for the estimates of spawning biomass.

The asymptotic coefficients of variation for spawning biomass and recruitment of cod and pollock (Figs 4.22 and 4.23, lower panels) are very similar for the "no predation" and base-case "with predation" methods, but the estimates from the "no predation" method are indicated to be notably less precise than those from the base-case "with predation" method for Atka mackerel.

4.4.5 General discussion

The "with predation" models are capable of adequately mimicking the data on which stock assessments are conventionally conducted as well as most of the data on diets based on stomach content sampling. However, and perhaps as expected, the fits to the catch, survey index and age-length composition information by the base-case "with predation" model are not as good as those of the "no predation" model, suggesting that there is some conflict between the diet data and the other sources of information on abundance. However, the estimates of recruitment and spawning biomass from the base-case "with predation" and "no predation" models are either equally precise (or the "with predation" model is more precise). However, it should be noted that the estimates of uncertainty are sensitive to the choice of the method for measuring parameter uncertainty.

The choices for some of the effective sample sizes are relatively arbitrary (e.g. a maximum of 20 for the diet composition data). While the values for the effective sample sizes cannot be treated as estimable parameters, consideration should be given in future work to applying the approach of McAllister and Ianelli (1999), perhaps implemented using the approach of MacCall (2003), to refine these effective sample sizes. Given the apparent conflict between the diet data and the remaining sources of information, it would not be unexpected that the results are sensitive to changes to the effective sample sizes (and hence the weights assigned to the various data sources).

The time-trajectories for quantities of management interest (viz. recruitment and spawning biomass) differ among choices for the feeding functional relationship (Figs 4.1 and 4.2), while Table 4.2 suggests that the data are capable of selecting among alternative feeding functional relationships. In fact, the model that leads to the estimates of the spawning biomass and age-0 abundance which differ the most from the base-case "with predation" model (Type I) also has the highest ΔAIC value (Table 4.2). Chapter 5 explores model selection further by means of simulation. The results from the “with predation” models also differ from those of the “no predation” model (e.g. Figs 4.13 and 4.14), with the extent of difference being greatest for Atka mackerel and least for Pacific cod.

Table 4.1. Estimable parameters for the Aleutian Island stocks of pollock, mackerel, and cod.

Parameter	Description	Number of parameters			
		Pollock	**Mackerel**	**Cod**	**Total**
M_k	natural mortality	1	1	Pre-specified (0.37)	2
α^r	prey size selectivity	$1^{\&}$	$1^{\&}$	$1^{\&}$	$3^{\&}$
β^r	prey size selectivity	$1^{\&}$	$1^{\&}$	$1^{\&}$	$3^{\&}$
$\ell\text{n}Q_o^r$	logarithm of “other prey” abundance	Pre-specified (9.32)	Pre-specified (9.32)	Pre-specified (9.32)	
v_k^r	predation parameter (all Types)	$4^{\&}$	$4^{\&}$	$4^{\&}$	$12^{\&}$
$\tilde{v}_k^r$	predation parameter (Eqns 4.3a-f)	3*	3*	3*	9*
$\tilde{\tilde{v}}_k^r$	predation parameter (Eqns 4.3c, 4.3d)	3*	3*	3*	9*
δ_k^r	predation parameter (Types 4.4b, 4.4e)	3*	3*	3*	9*
ω^r	determines age-dependence of predation	$1^{\&}$	$1^{\&}$	$1^{\&}$	$3^{\&}$
$\tilde{\omega}^r$	determines age-dependence of predation	$1^{\&}$	$1^{\&}$	$1^{\&}$	$3^{\&}$
h_k	steepness of the stock-recruitment relationship	Pre-specified (0.6)	Pre-specified (0.8)	Pre-specified (0.6)	
$R_{0.k}$	mean unfished recruitment	1	1	1	3
$\varepsilon_{k,y}^R$	logarithm of year-specific deviation in recruitment	42	42	34	118
$\sigma_{\text{R},k}$	extent of recruitment variability	Pre-specified (0.5)	Pre-specified (0.6)	Pre-specified (0.6	
$\eta_{y,a}^{k,f}$	logarithm of fishery selectivities	12	270	20	302
q_k^s	survey catchability coefficient	Pre-specified (1)	Pre-specified (1)	Pre-specified (1)	
$\eta_{k,a}^s$	logarithm of survey selectivites	10	10	10	30
$\varepsilon_{k,y}^f$	logarithm of year-specific deviation in fishing mortality	24	25	63	112
μ_k^f	logarithm of average fishing mortality over all years	1	1	3	5
Total$^+$		99	358	139	596^+

* if estimated.
& all “with predation models”
\+ linear model

Table 4.2. Comparison of the seven predation models based on AIC.

	Model						
	Type I	Type II	Type III	Predator interference	Predator pre-emption	Hassel-Varley	Ecosim-like
# parameters	596	605	614	614	614	614	605
AIC	8794.56	8326.82	8426.34	8322.08	7999.96	8405.96	8302.90
ΔAIC	794.60	326.86	426.38	322.12	0.00	406.00	302.94

Table 4.3. The contribution of the various data sources to the objective function (expressed as differences from the AIC-selected predation model).

	Objective function component							
Model	Catch biomass	Fishery composition	Survey Index	Survey composition	Daily ration	Diet (weight composition)	Diet (length composition)	Penalties
Type I	4.56	14.10	15.25	67.55	36.83	227.24	34.52	15.24
Type II	1.01	-33.35	4.40	-14.52	13.01	188.79	27.49	-14.40
Type III	6.40	21.53	15.89	44.08	47.97	104.93	-126.77	99.17
Predator interference	0.54	-25.82	0.21	-20.56	9.33	177.58	42.78	-23.00
Predator pre-emption	***0.00***	***0.00***	***0.00***	***0.00***	***0.00***	***0.00***	***0.00***	***0.00***
Hassel-Varley	2.22	-29.93	9.36	-11.03	10.46	184.70	47.40	-10.18
Ecosim-like	1.24	7.58	2.21	-3.38	14.66	80.75	25.44	31.97
No predation	1.57	-37.54	3.81	-19.08	N/A	N/A	N/A	

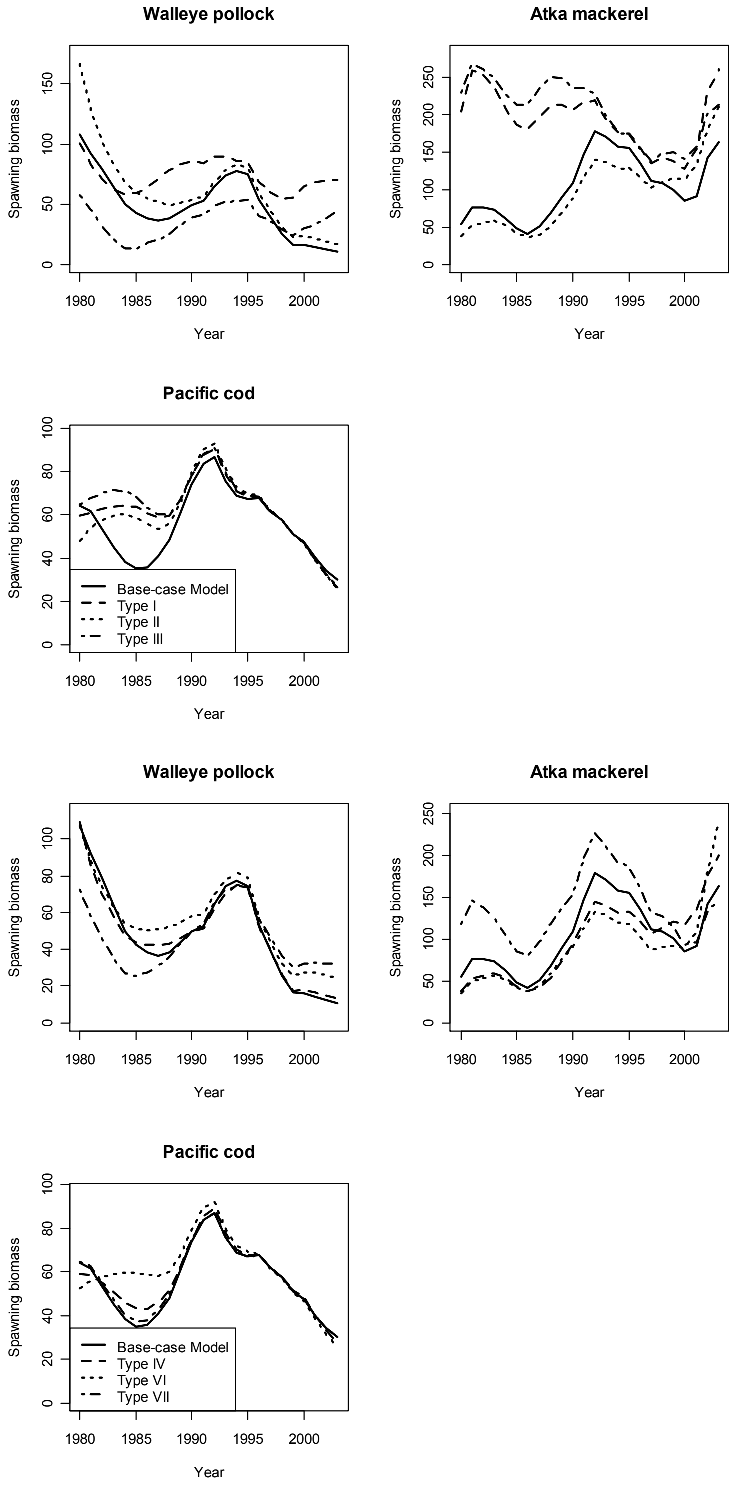


Figure 4.1. Time-trajectories of spawning biomass (1980-present) for the base-case “with predation” model and for the six alternative feeding functional relationships.

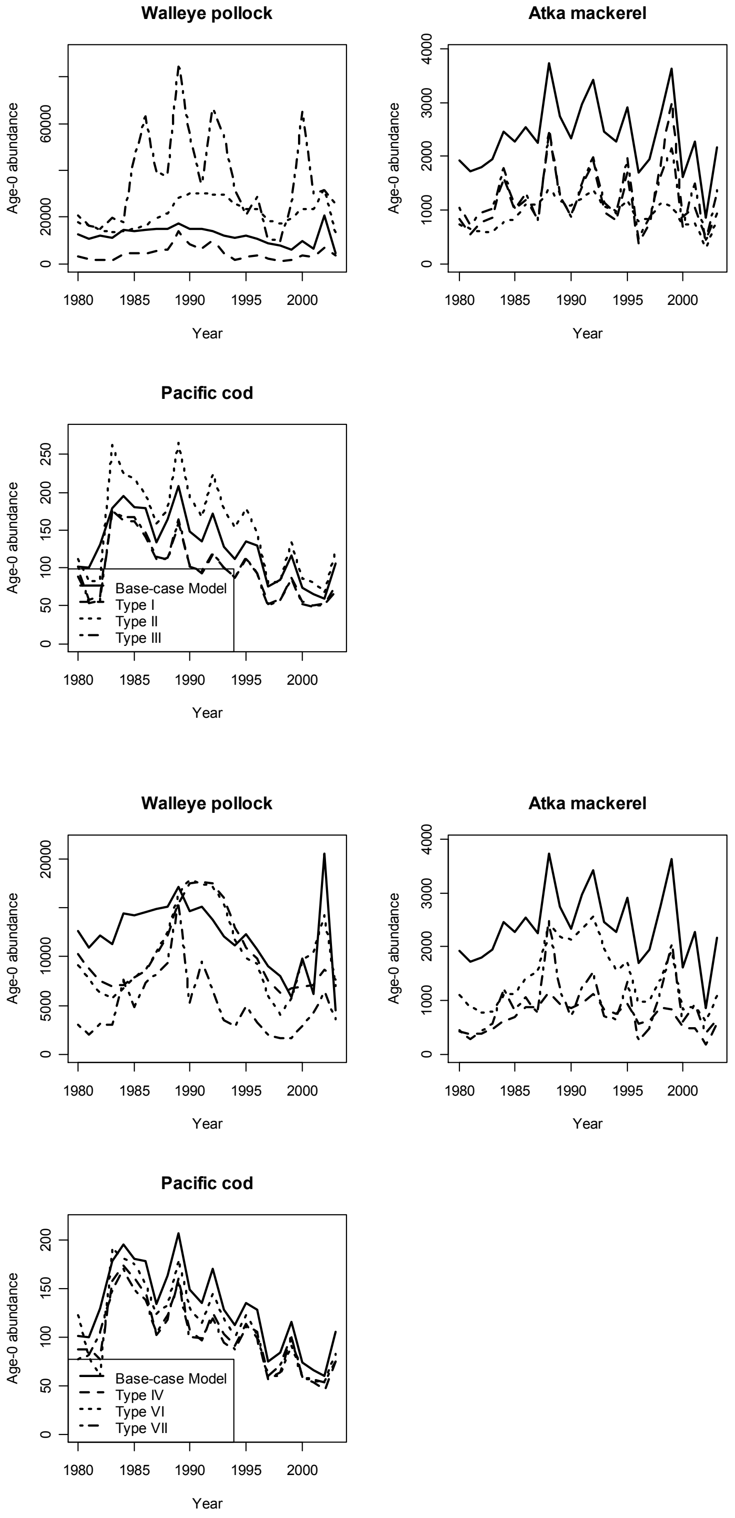


Figure 4.2. Time-trajectories of age-0 abundance (1980-present) for the base-case "with predation" model and for the six alternative feeding functional relationships.

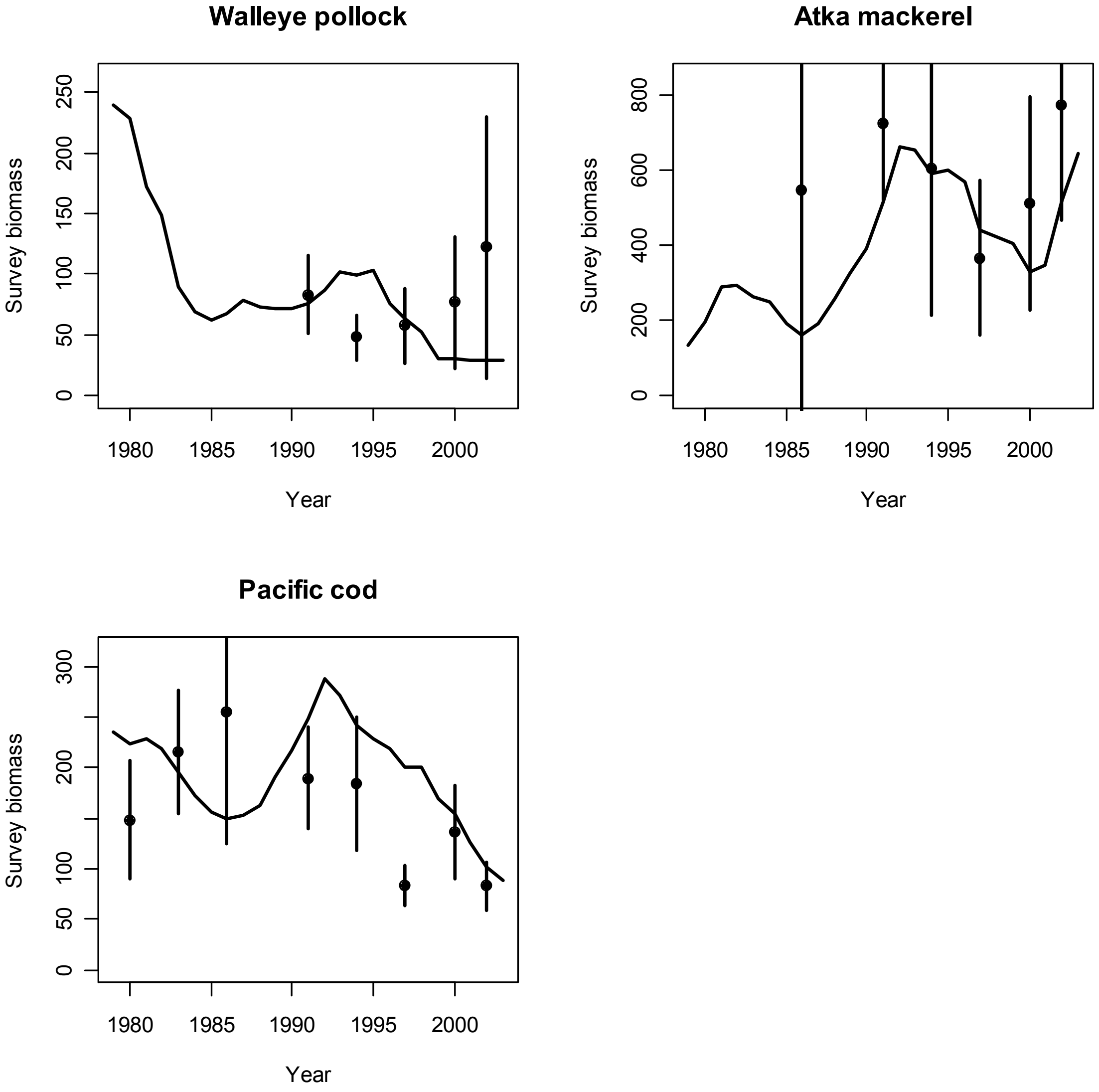


Figure 4.3. Fit of the base-case "with predation" model to the survey indices of abundance.

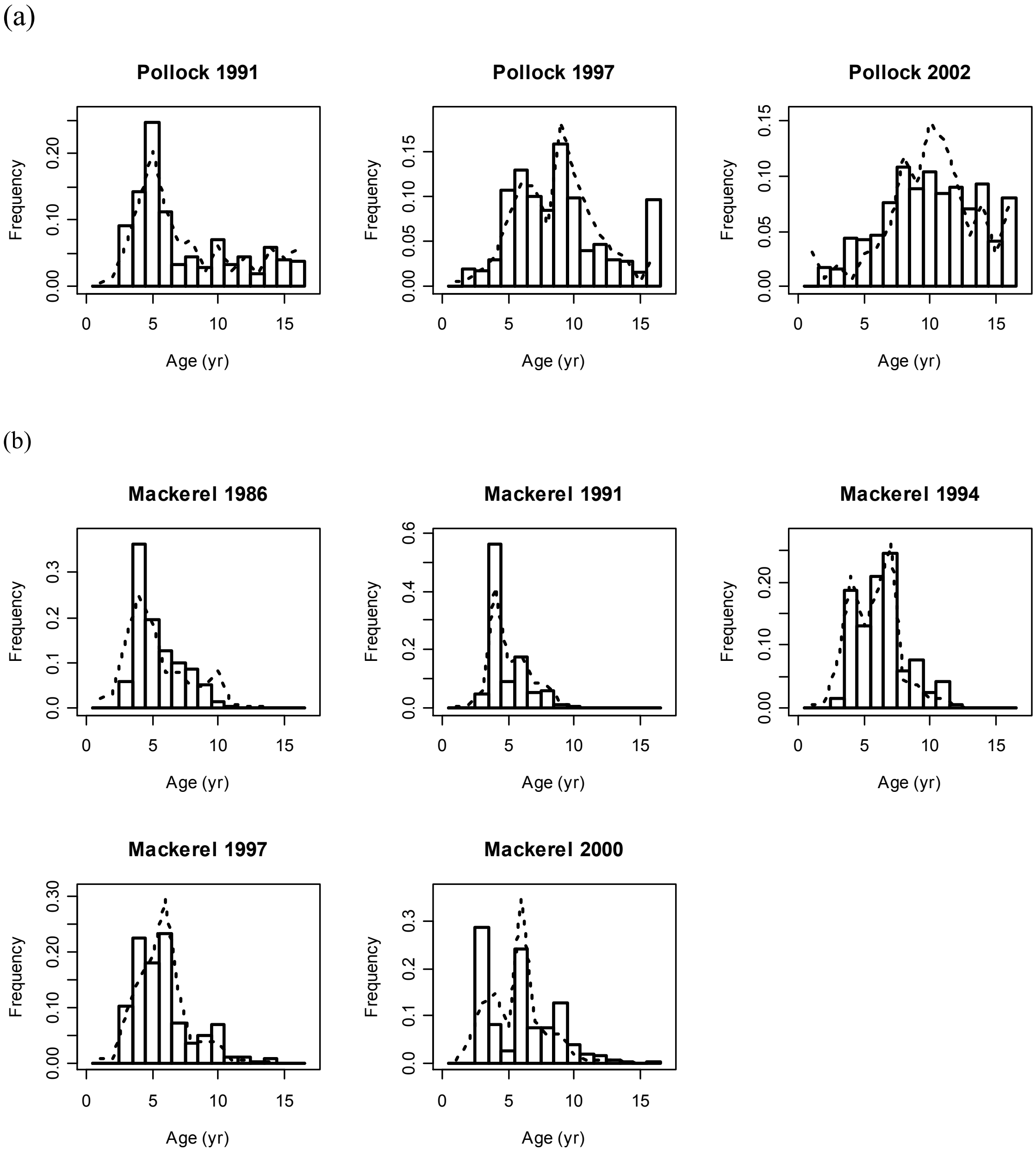
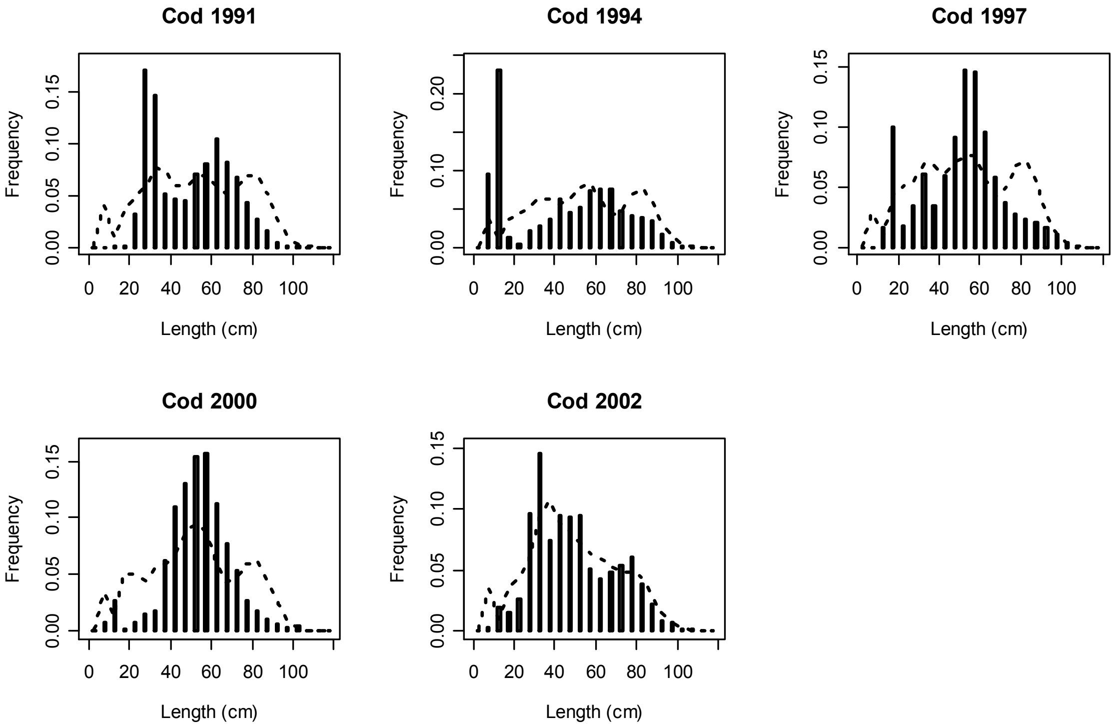


Figure 4.4. Fit of the base-case “with predation” model to the survey age-/length-composition data.

(c)



(Figure 4.4 Continued)

(a)

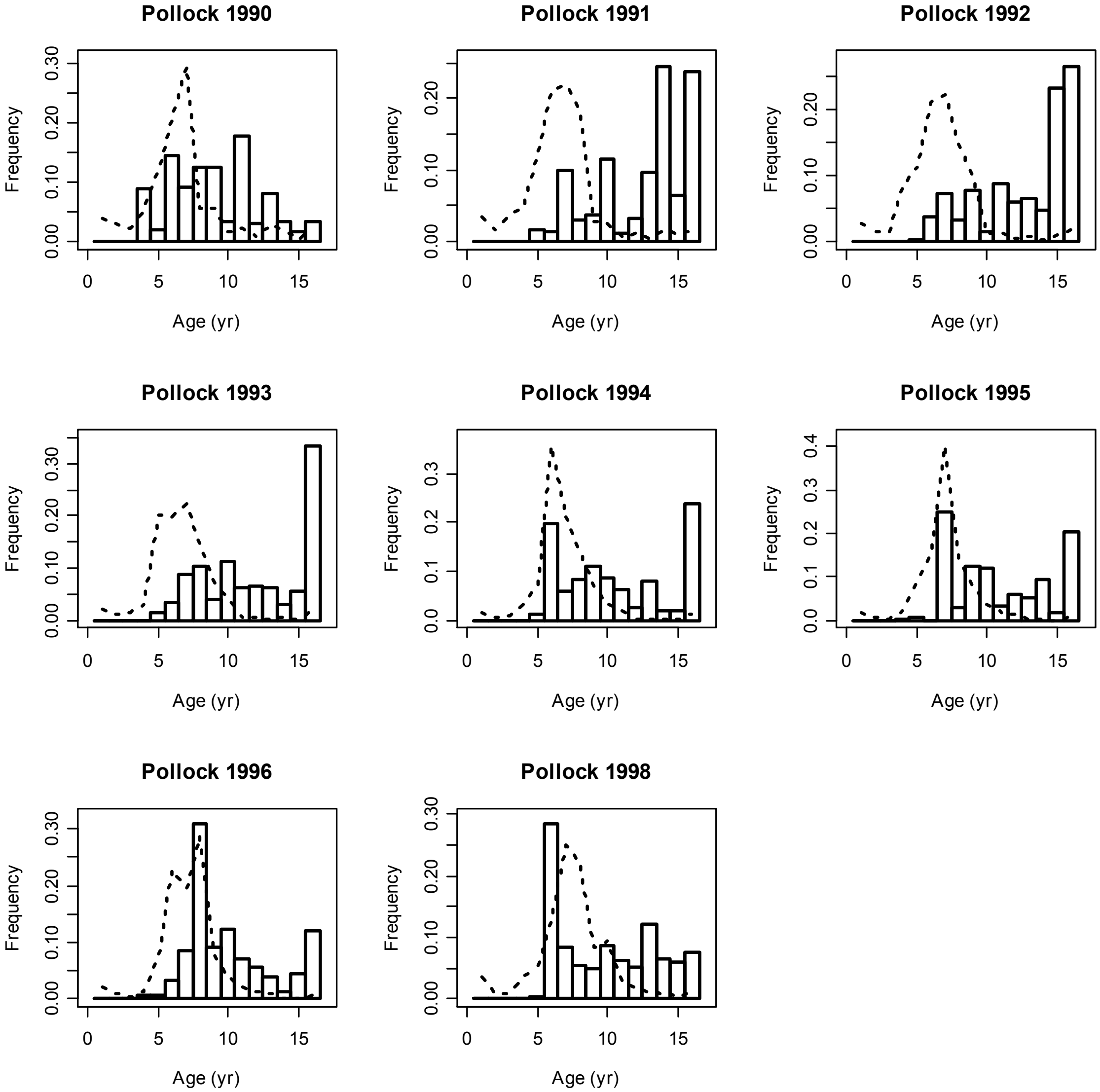
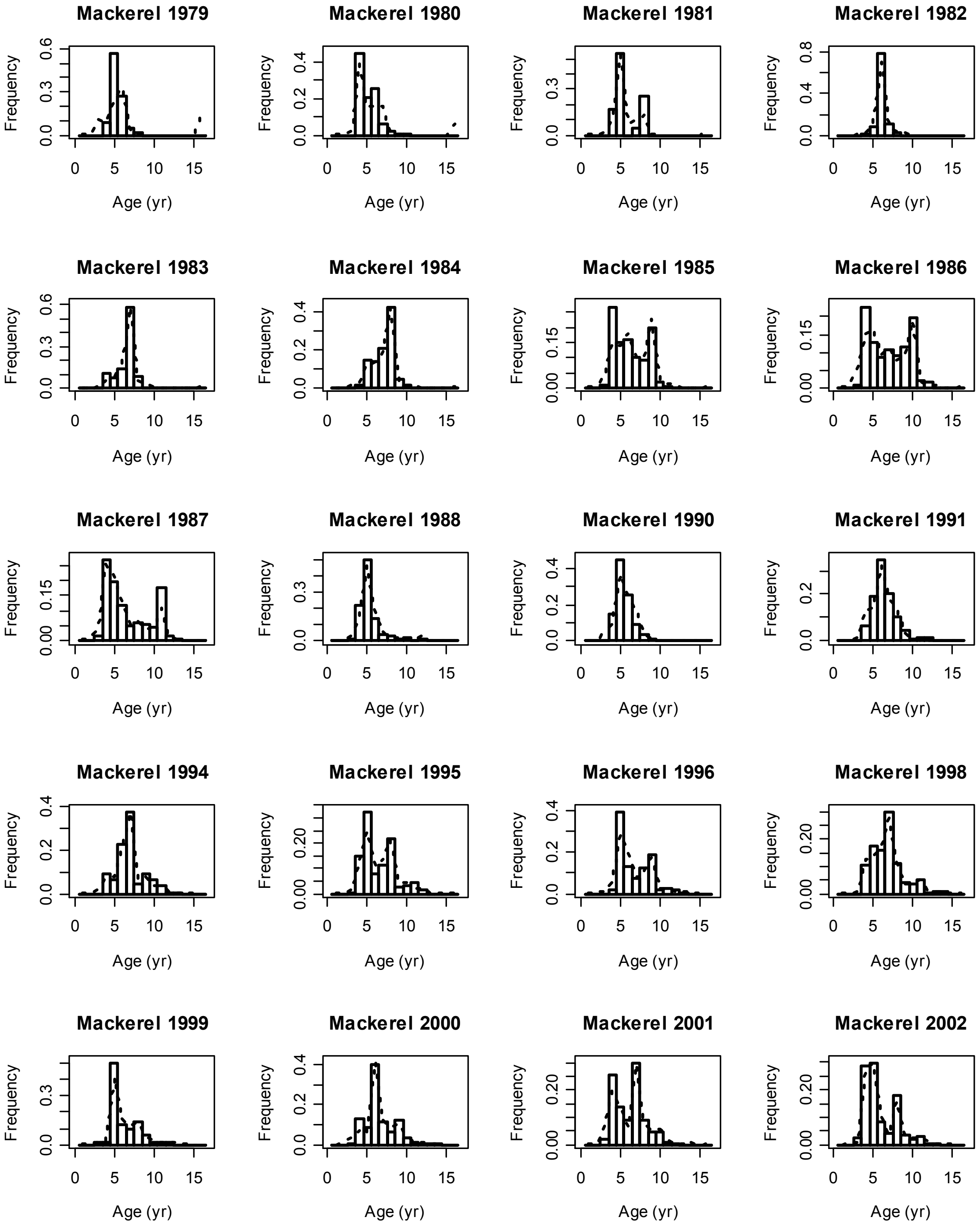


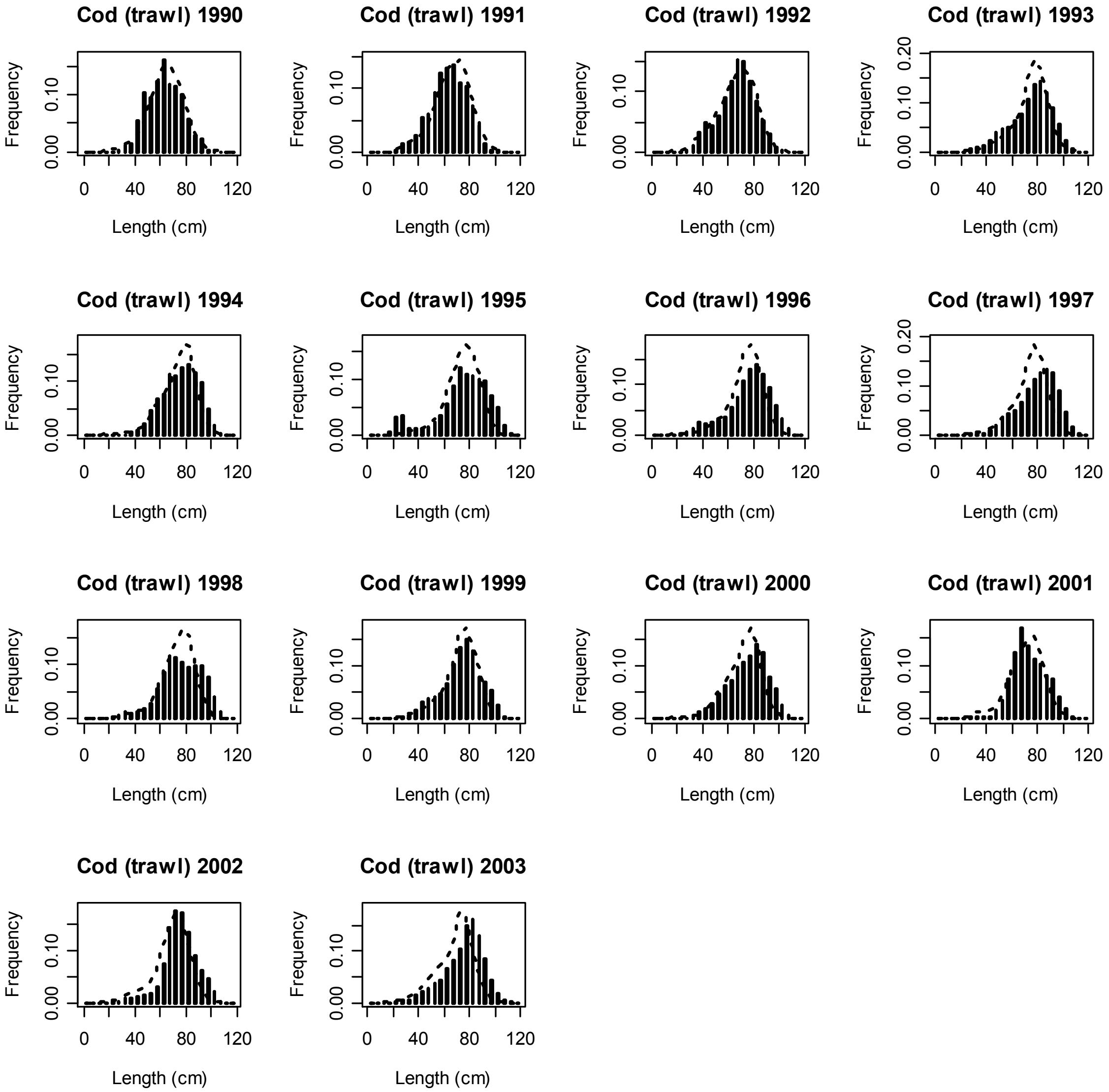
Figure 4.5. Fit of the base-case “with predation” model to the fishery age-/length-composition data.

(b)



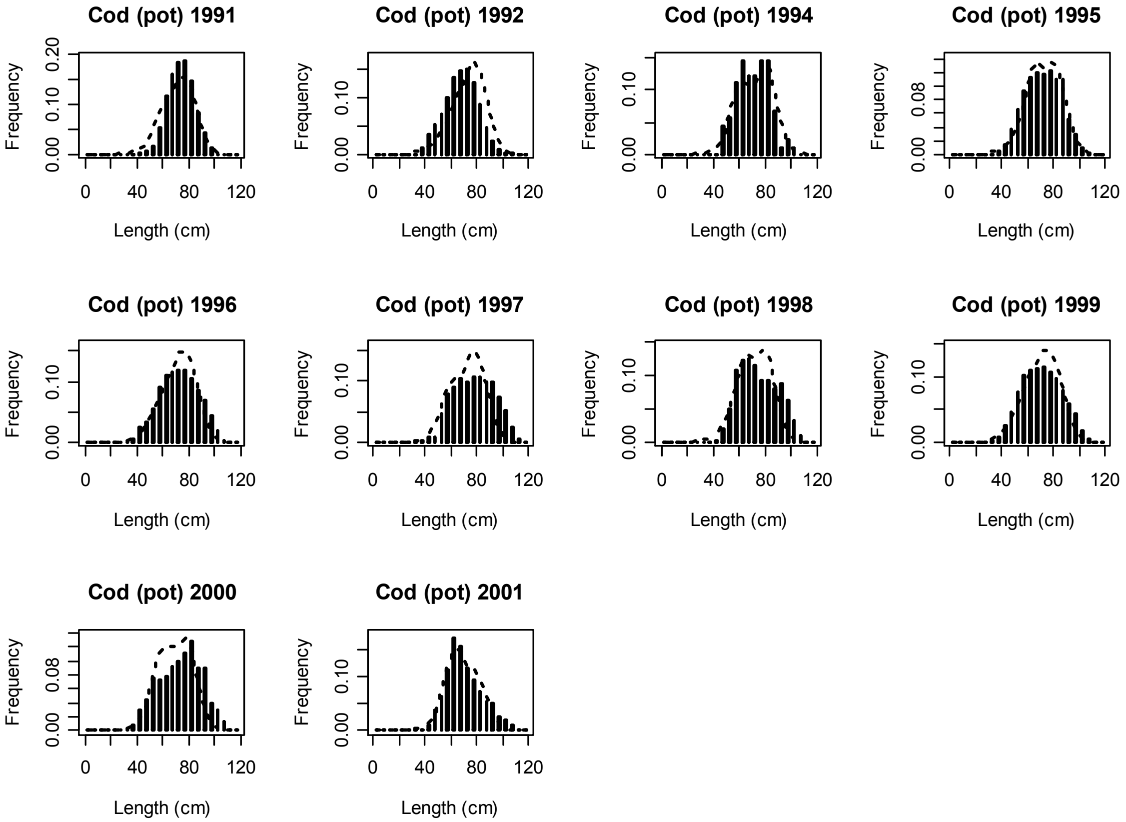
(Figure 4.5 continued)

(c)



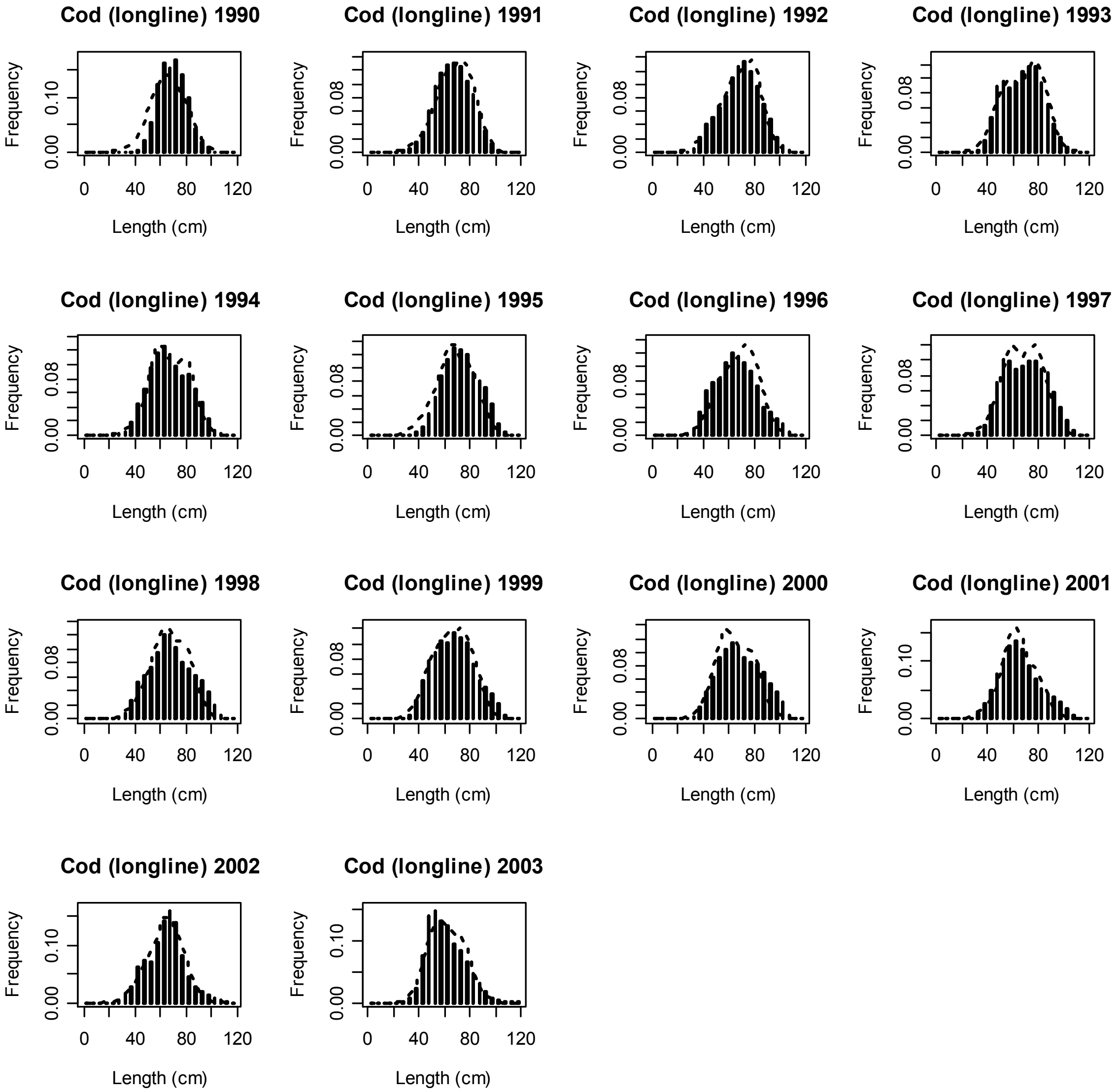
(Figure 4.5 continued)

(d)



(Figure 4.5 continued)

(e)



Cod (longline) 1990

Cod (longline) 1991

Cod (longline) 1992

Cod (longline) 1993

Cod (longline) 1994

Cod (longline) 1995

Cod (longline) 1996

Cod (longline) 1997

Cod (longline) 1998

Cod (longline) 1999

Cod (longline) 2000

Cod (longline) 2001

Cod (longline) 2002

Cod (longline) 2003

(Figure 4.5 continued)

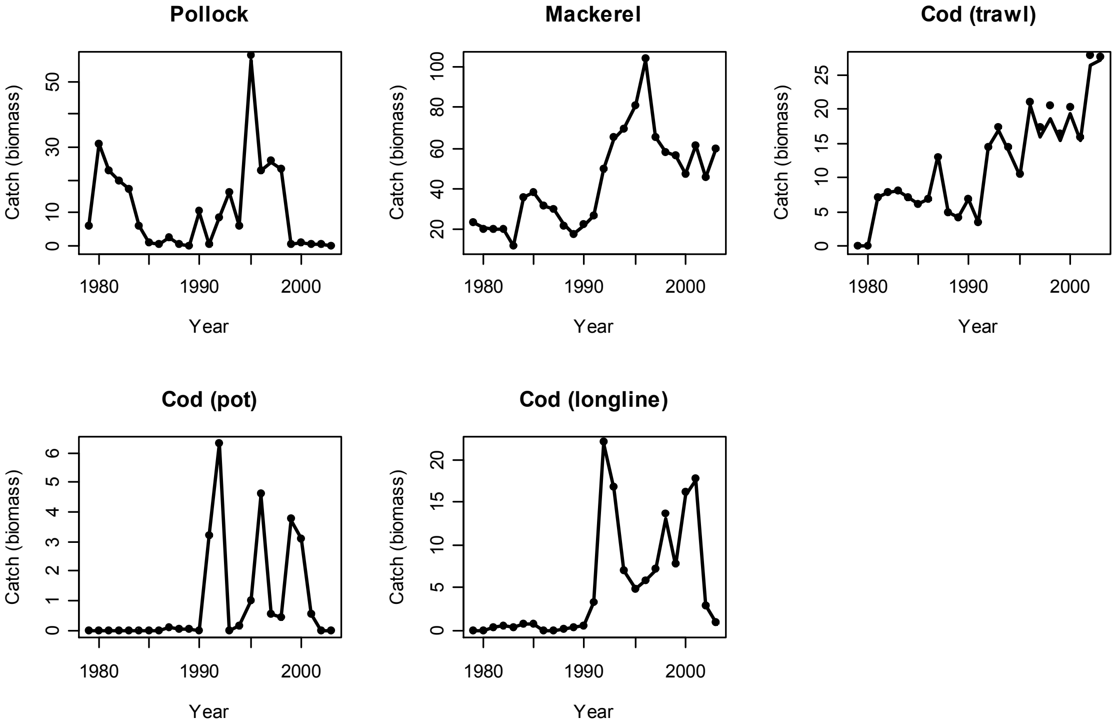


Figure 4.6. Fit of the base-case “with predation” model to the fishery catch biomass data.

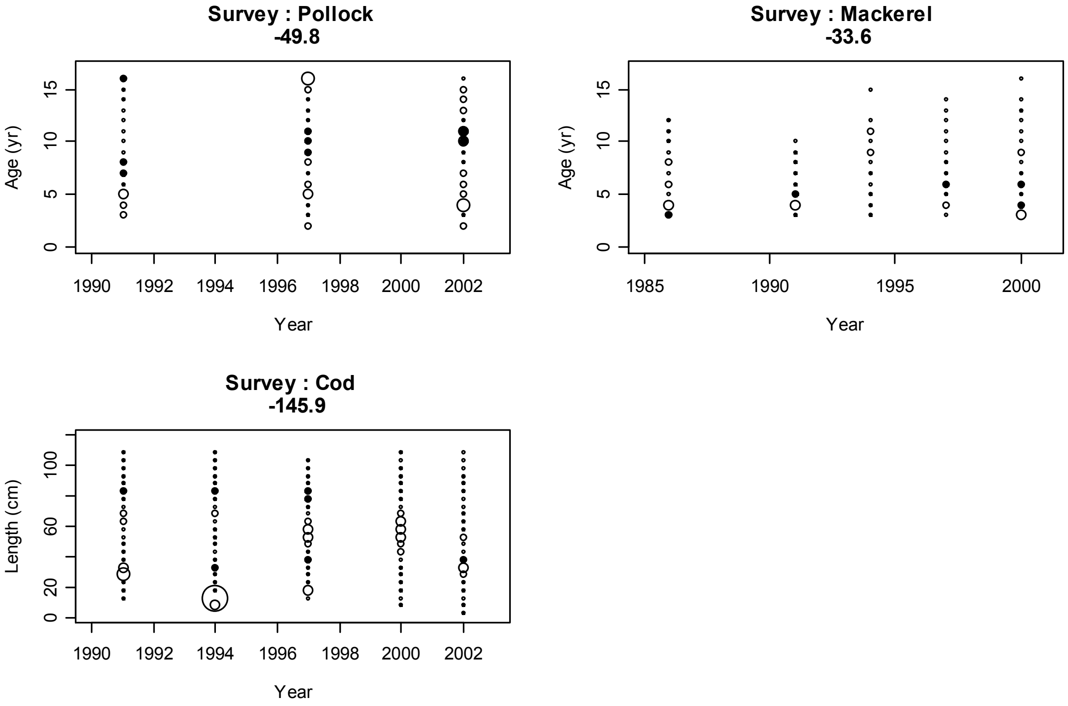


Figure 4.7. Bubble plots summarizing the fits to the survey age-/length-composition data. The numbers at the top of each plot indicate the contribution of the data source to the objective function.

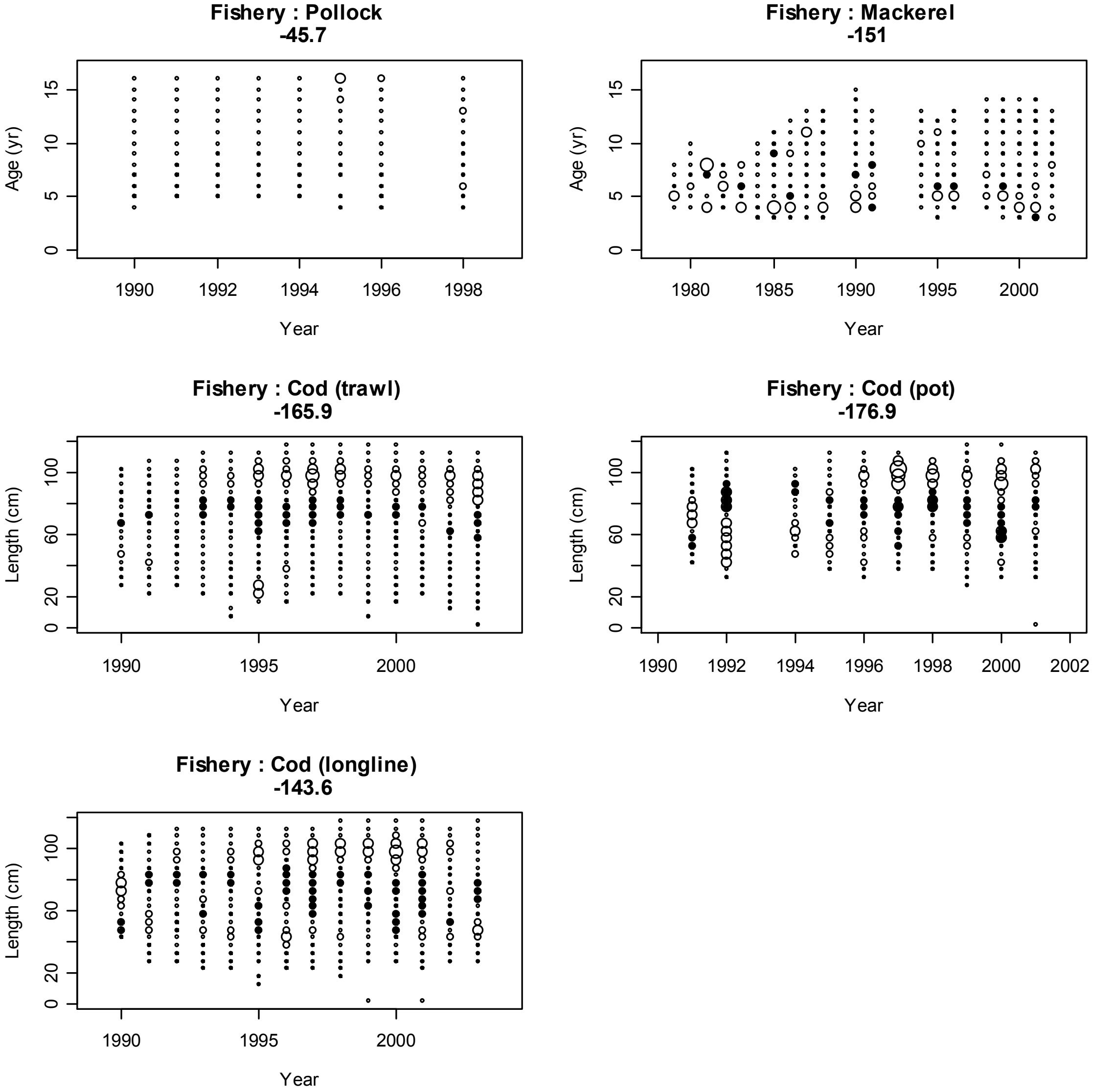


Figure 4.8. Bubble plots summarizing the fits to the fishery age-/length-composition data. The numbers at the top of each plot indicate the contribution of the data source to the objective function.

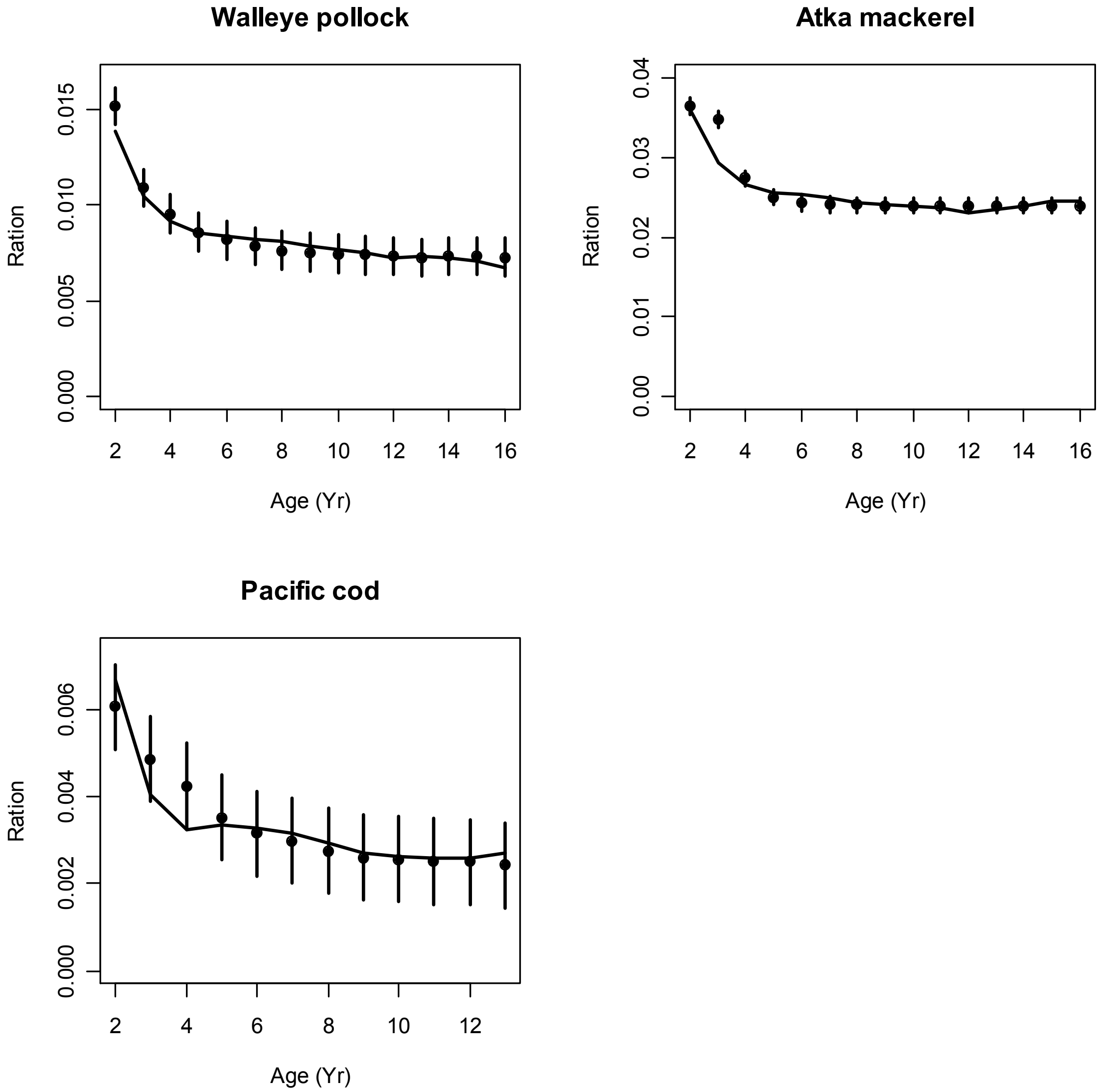


Figure 4.9. Fit of the base-case “with predation” model to the information on daily rations.

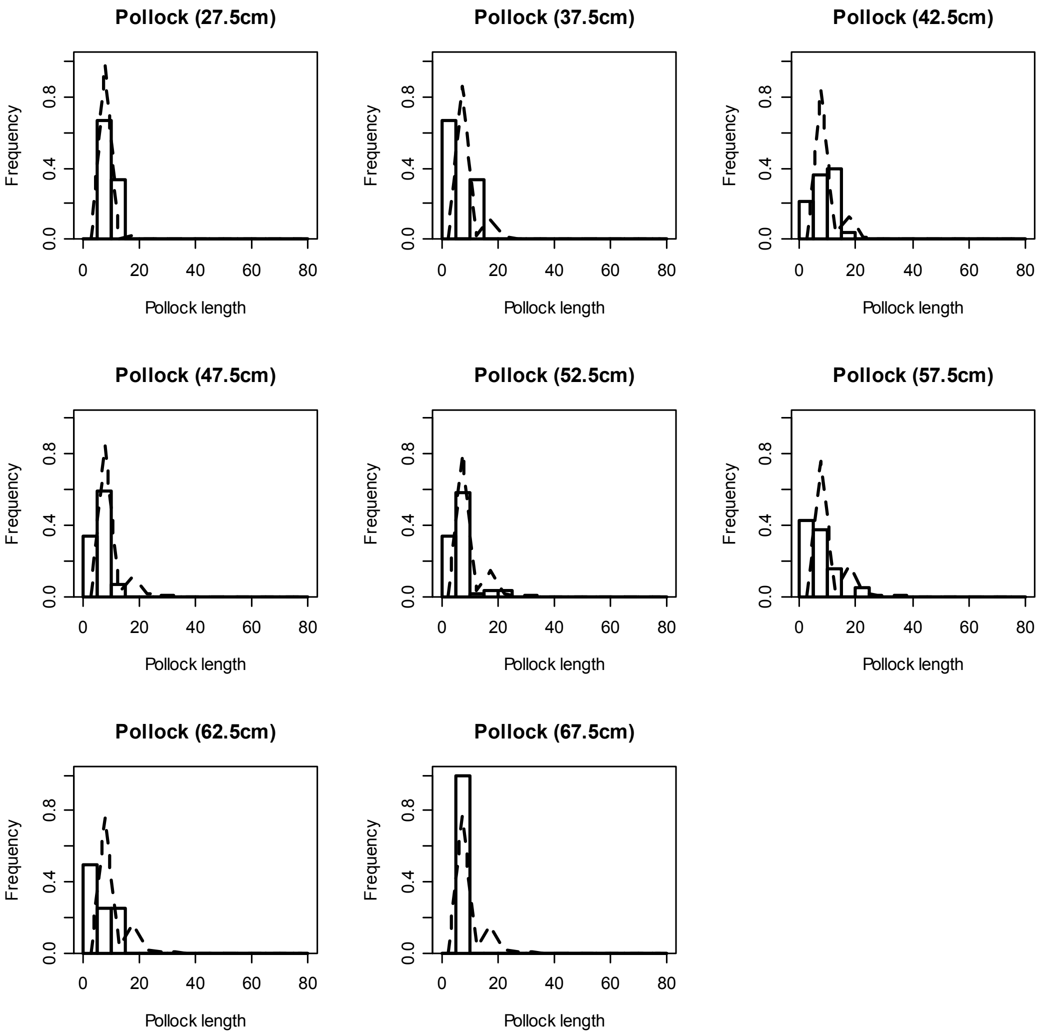


Figure 4.10(a). Fit of the base-case “with predation” model to the length-composition of walleye pollock in the diet of walleye pollock in various length-classes.

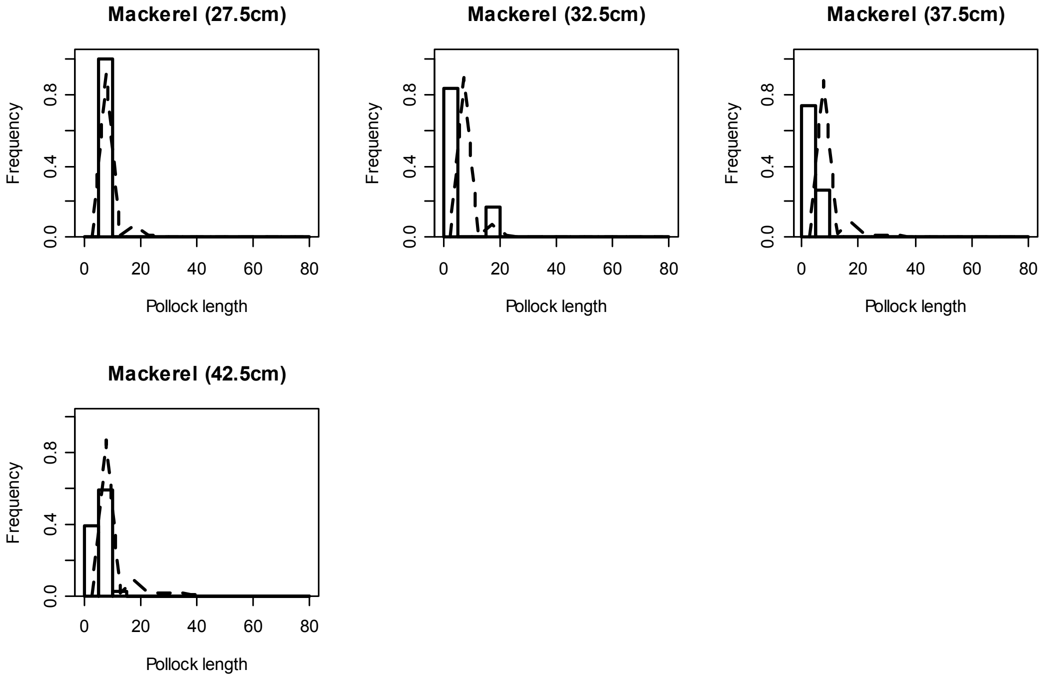


Figure 4.10(b). Fit of the base-case “with predation” model to the length-composition of walleye pollock in the diet of Atka mackerel in various length-classes.

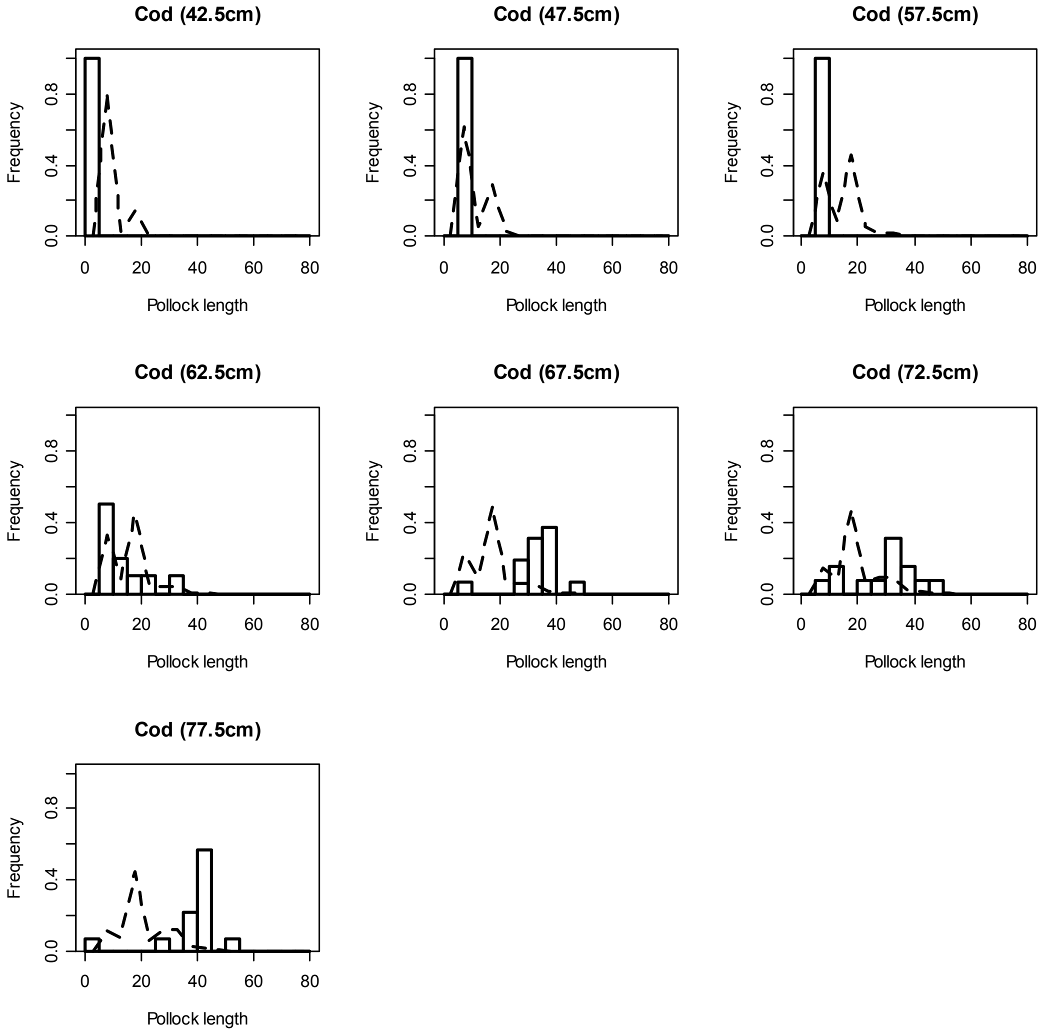


Figure 4.10(c). Fit of the base-case “with predation” model to the length-composition of walleye pollock in the diet of Pacific cod in various length-classes.

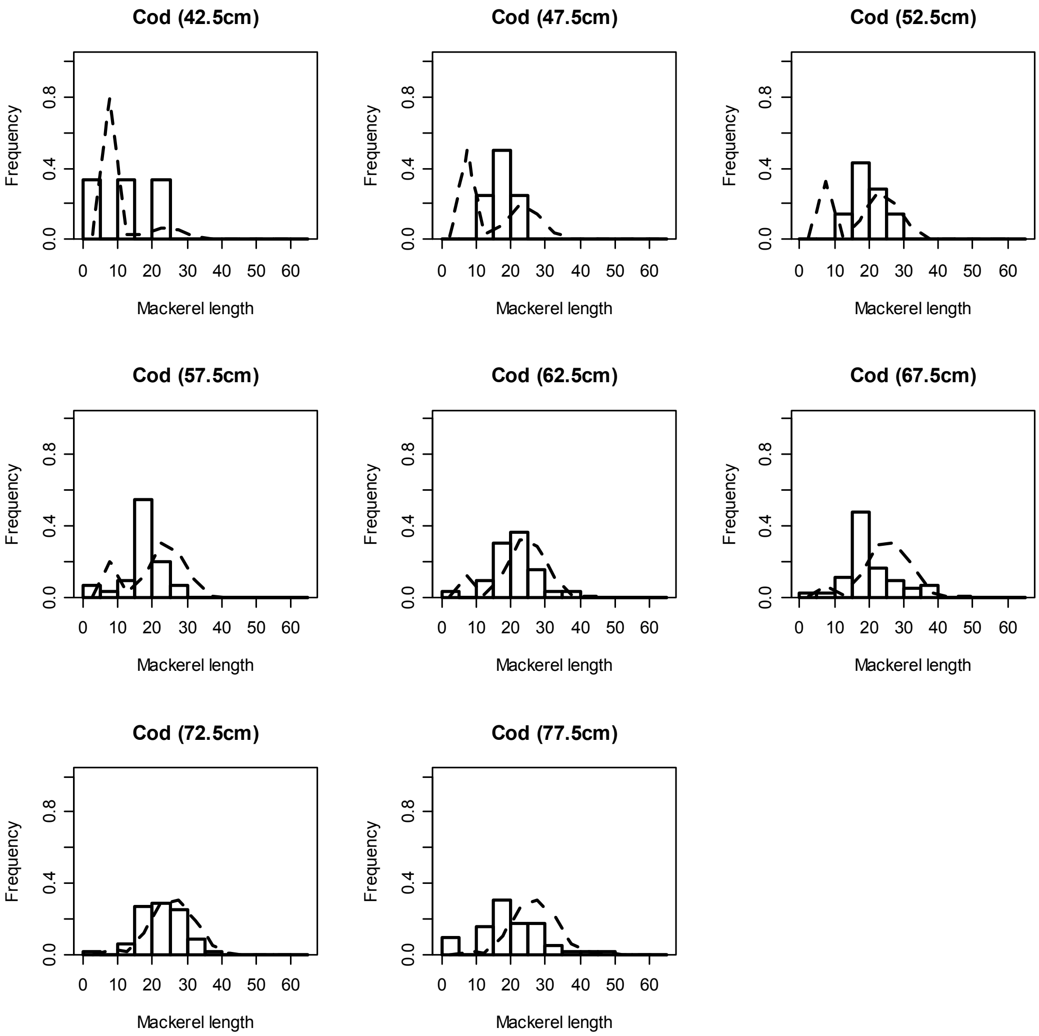


Figure 4.10(d). Fit of the base-case “with predation” model to the length-composition of Atka mackerel in the diet of Pacific cod in various length-classes.

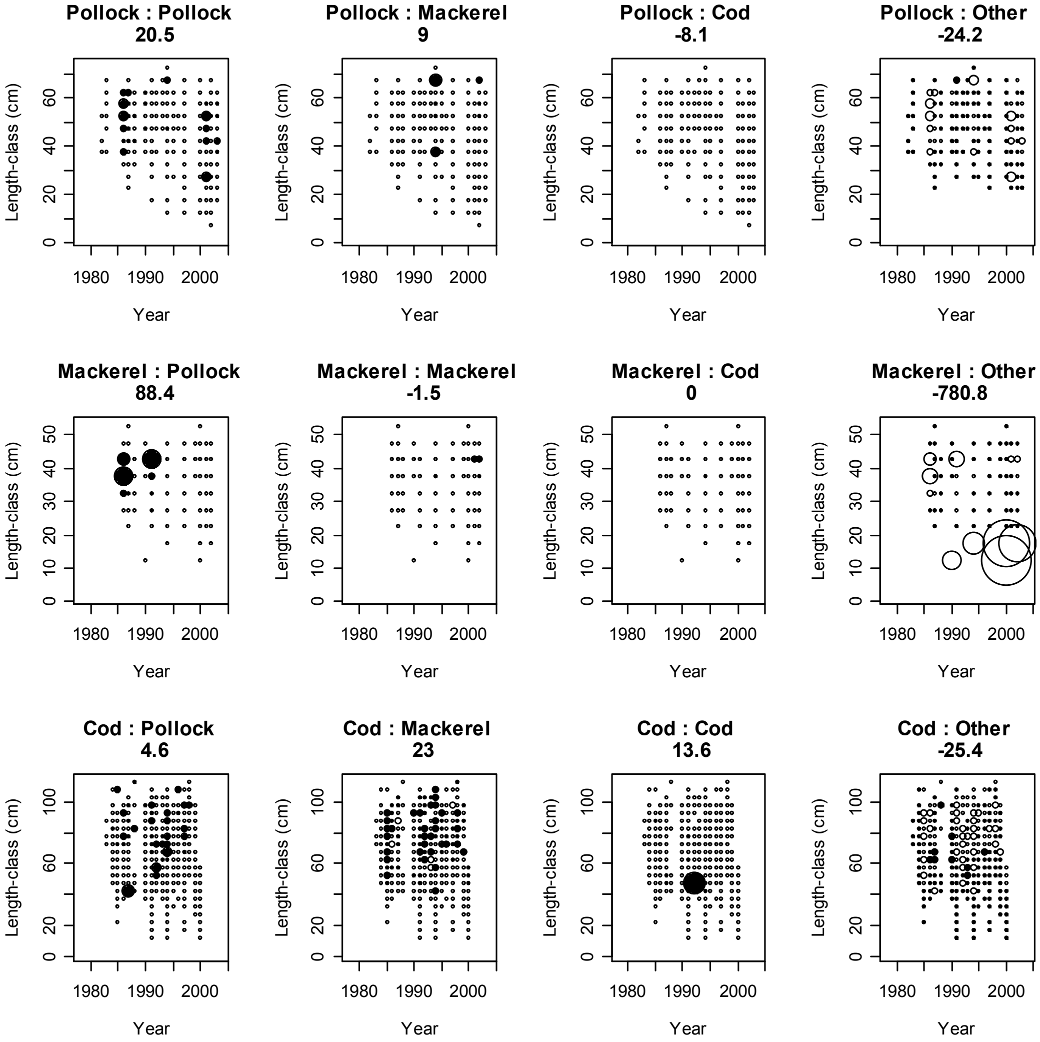


Figure 4.11. Bubble plots summarizing the fits to the diet weight-composition data. The numbers at the top of each plot indicate the contribution of the data source to the objective function.

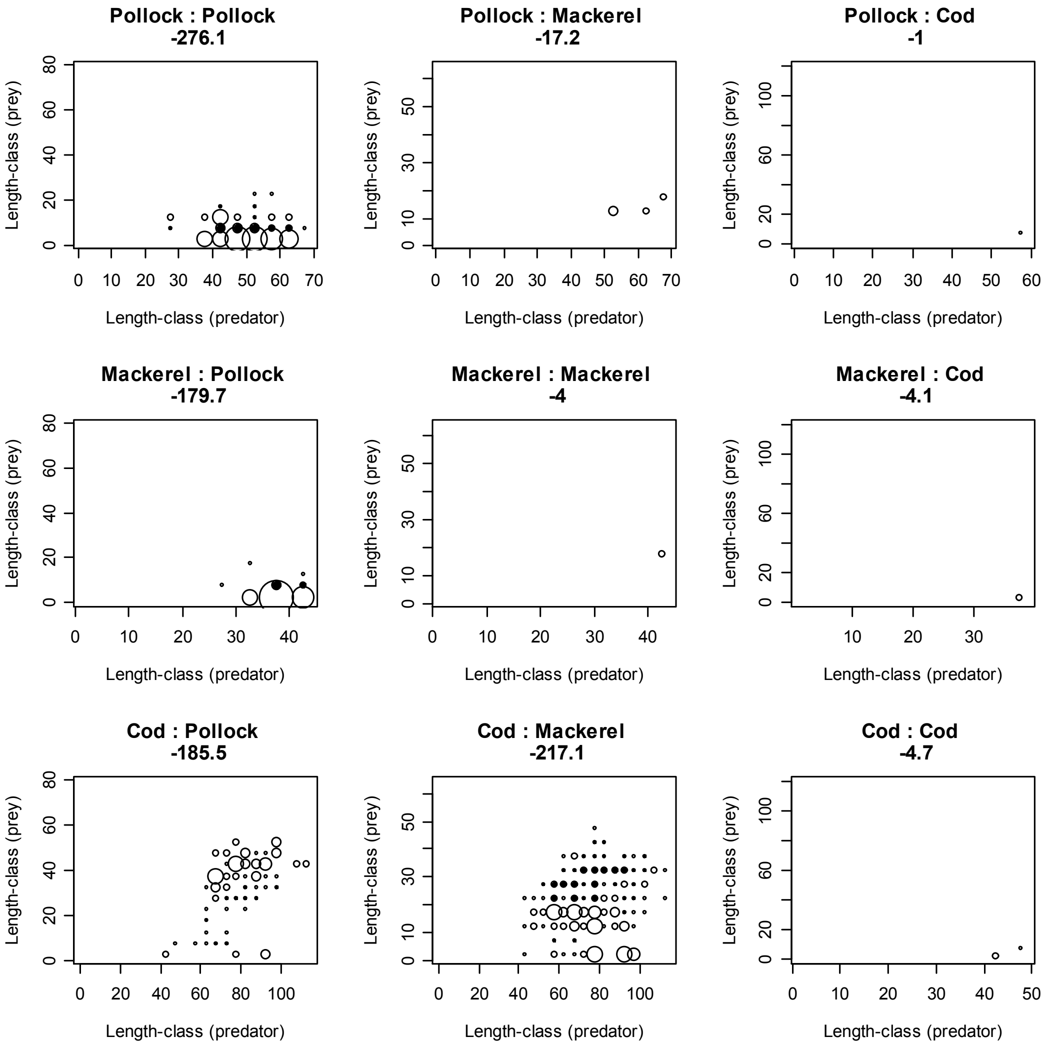


Figure 4.12. Bubble plots summarizing the fits to the diet length-composition data. The numbers at the top of each plot indicate the contribution of the data source to the objective function.

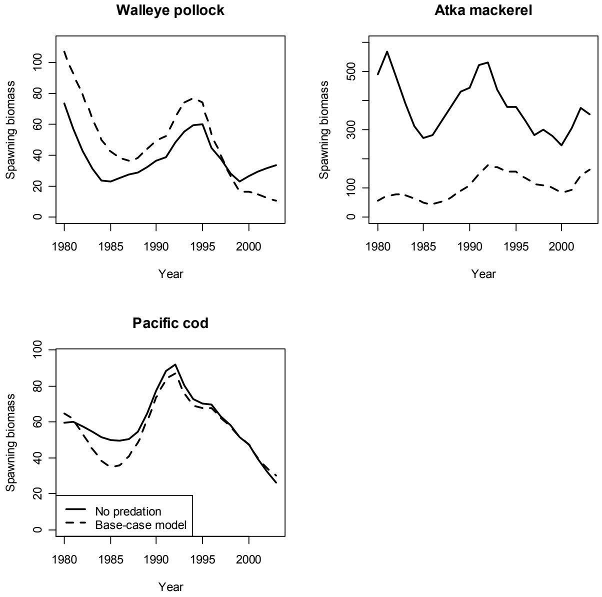


Figure 4.13. Time-trajectories of spawning biomass (1980-present) for the "no predation" and the base-case "with predation" models.

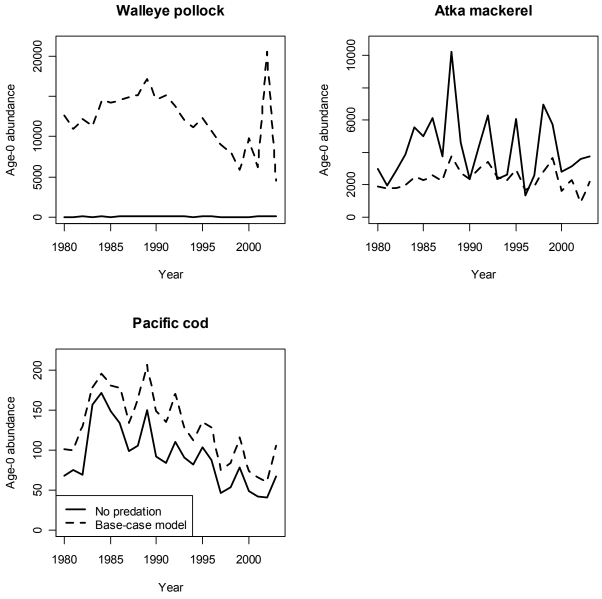
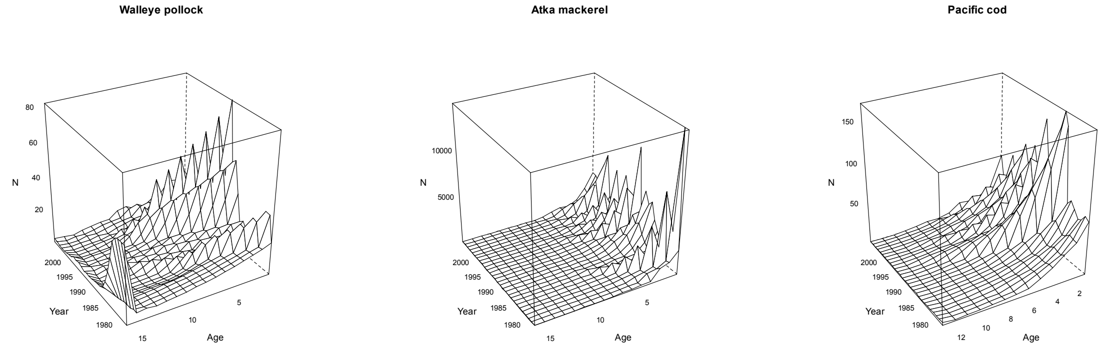


Figure 4.14. Time-trajectories of age-0 abundance (1980-present) for the "no predation" and the base-case "with predation" models.

(a) “No predation”



(b) “With predation”

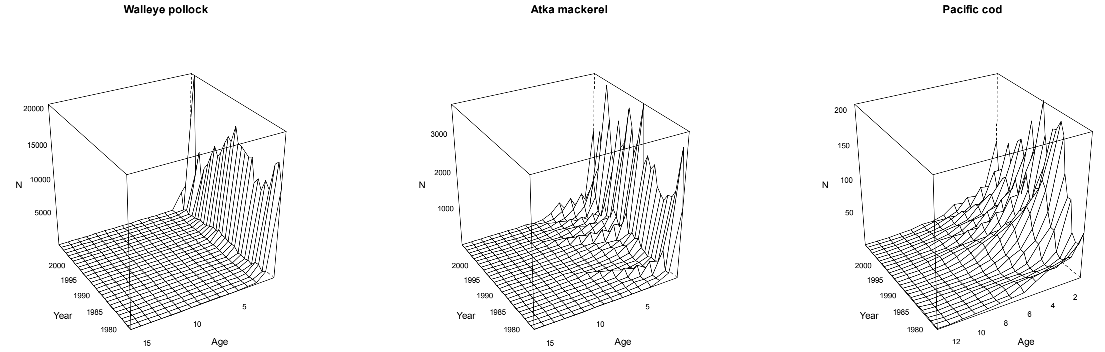
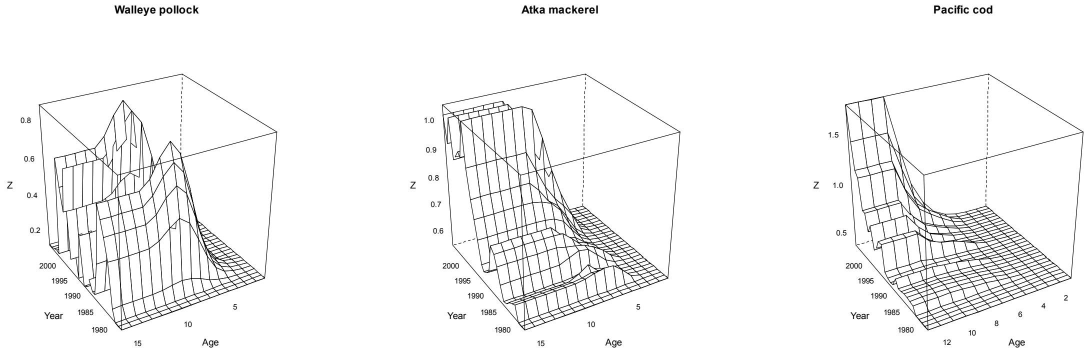


Figure 4.15. Numbers-at-age by species (1976-present) for the “no predation” and the base-case “with predation” models.

(a) “No predation”



(b) “With predation”

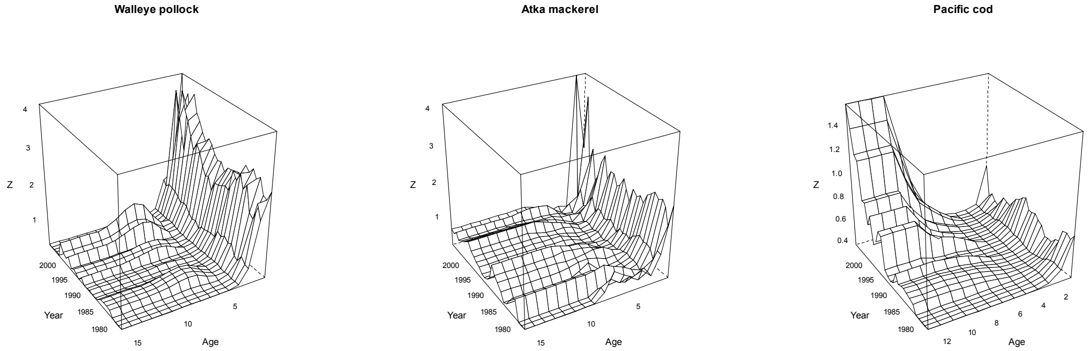


Figure 4.16. Total mortality-at-age by species (1976-present) for the “no predation” and the base-case “with predation” models.

(a) “No predation”

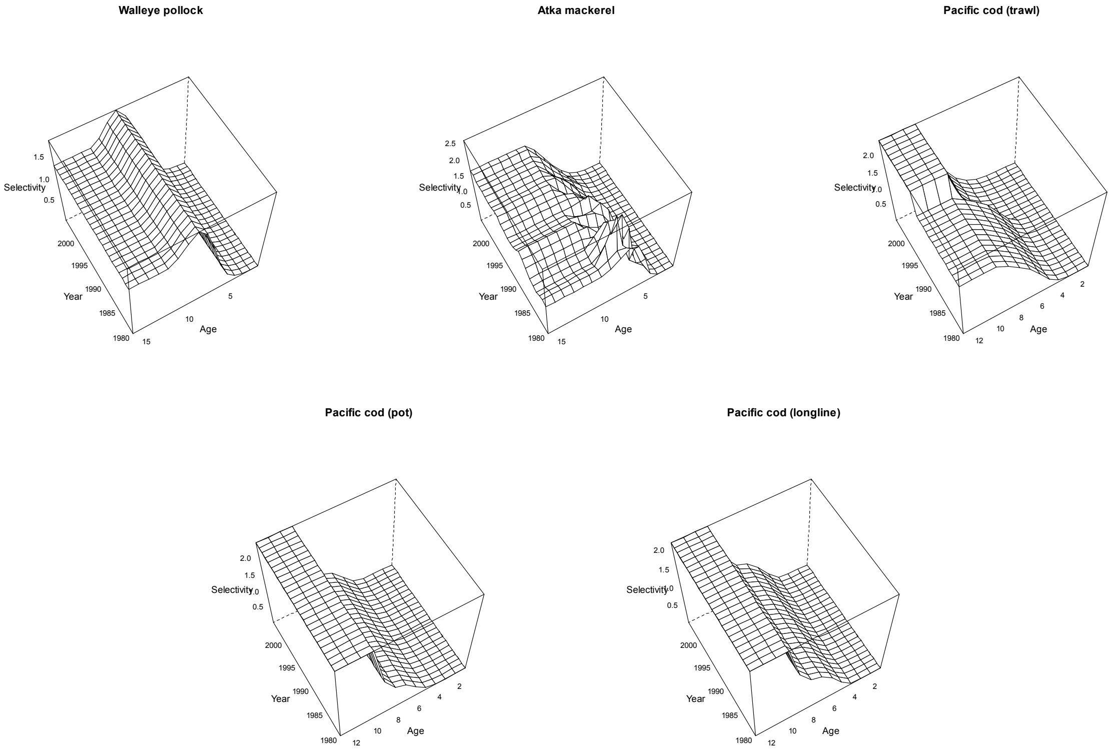
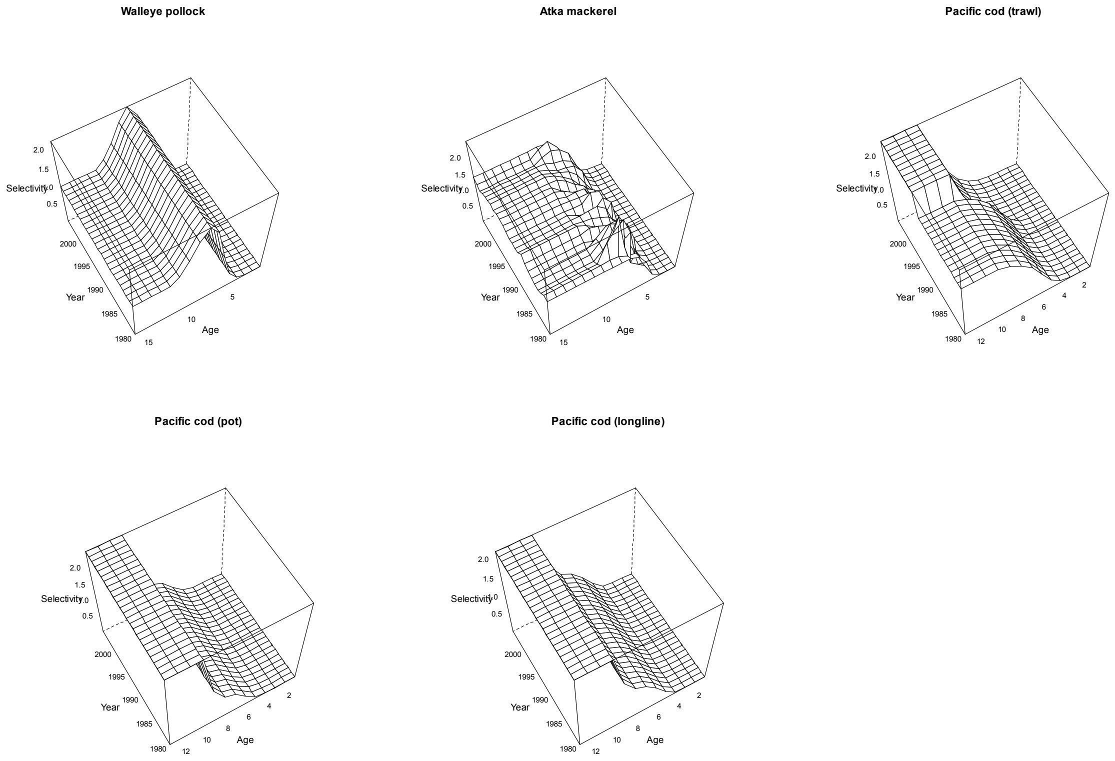


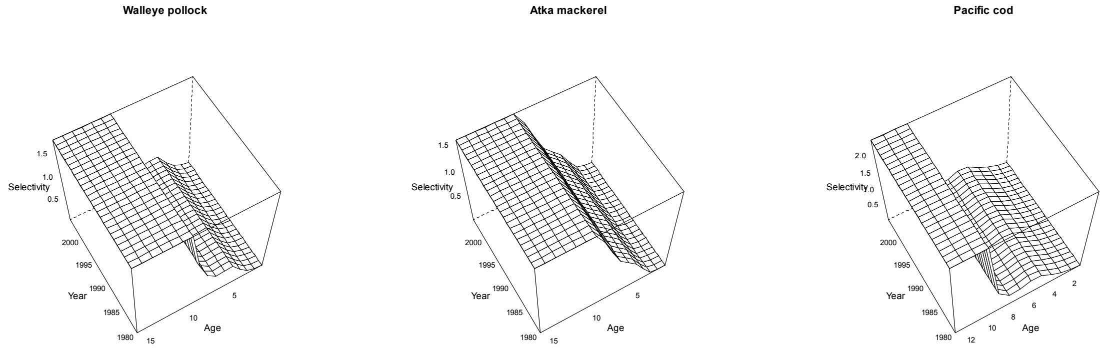
Figure 4.17. Fishery selectivity-at-age by species (1979-present) for the “no predation” and the base-case “with predation” models.

(b) “With predation”



(Figure 4.17 Continued)

(a) “No predation”



(b) “With predation”

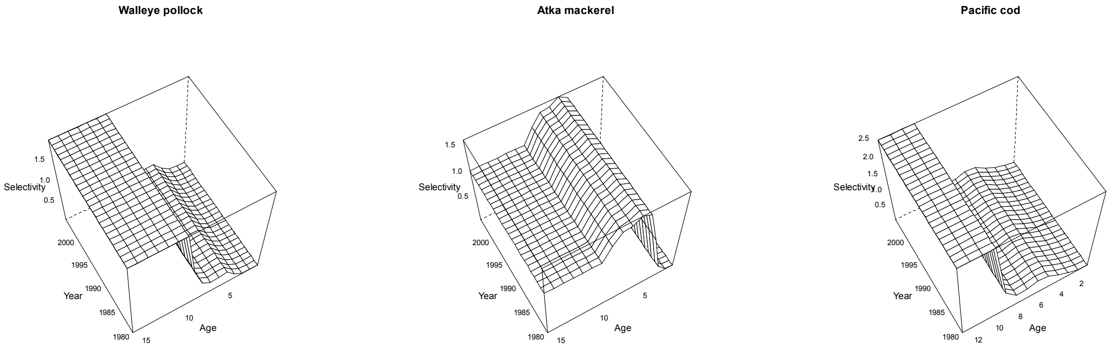
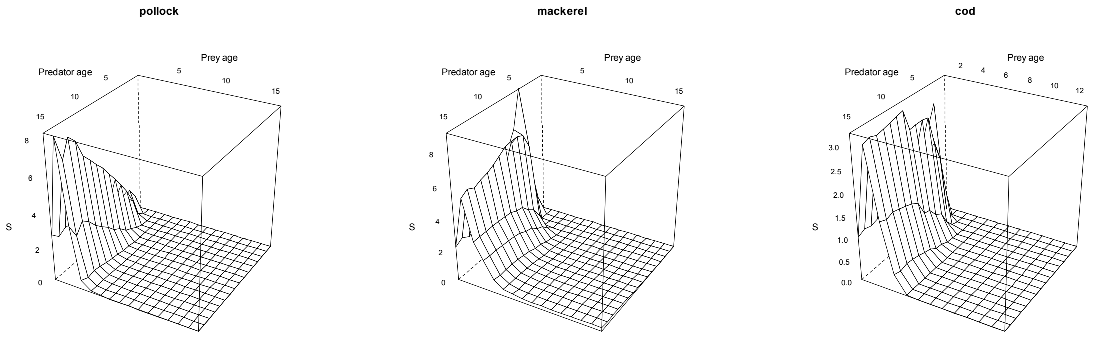
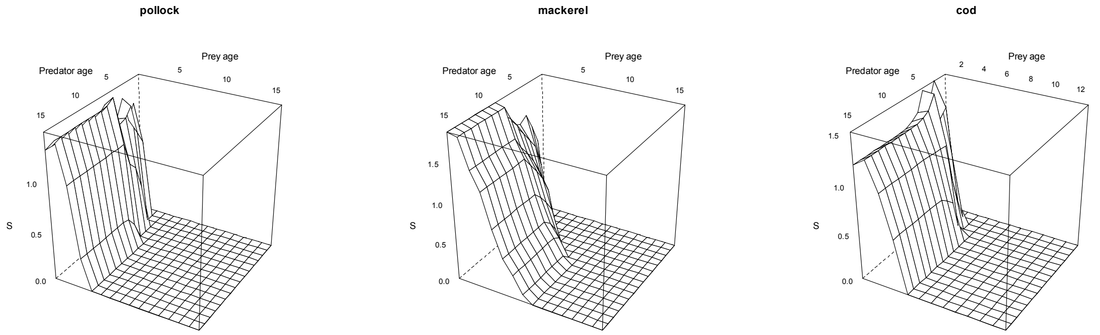


Figure 4.18. Survey selectivity-at-age by species (1979-present) for the “no predation” and the base-case “with predation” models.

(a) Pollock as predator



(b) Mackerel as predator



(c) Cod as predator

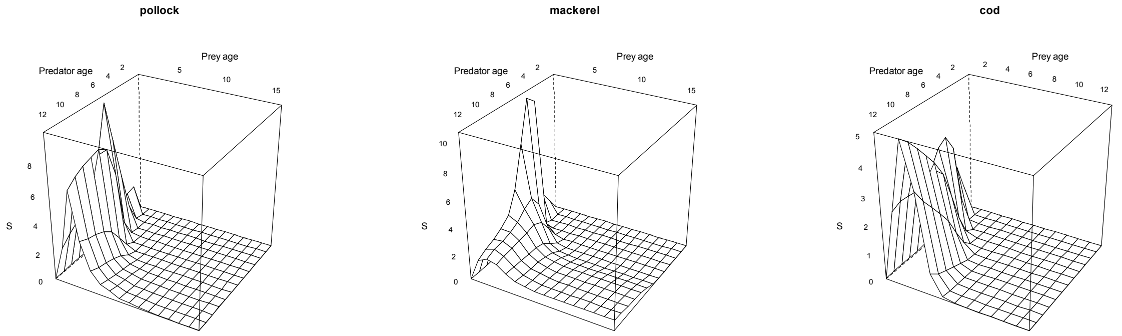


Figure 4.19. Predator selectivity-at-age by predator and prey species based on the predator pre-emption “with predation” model.

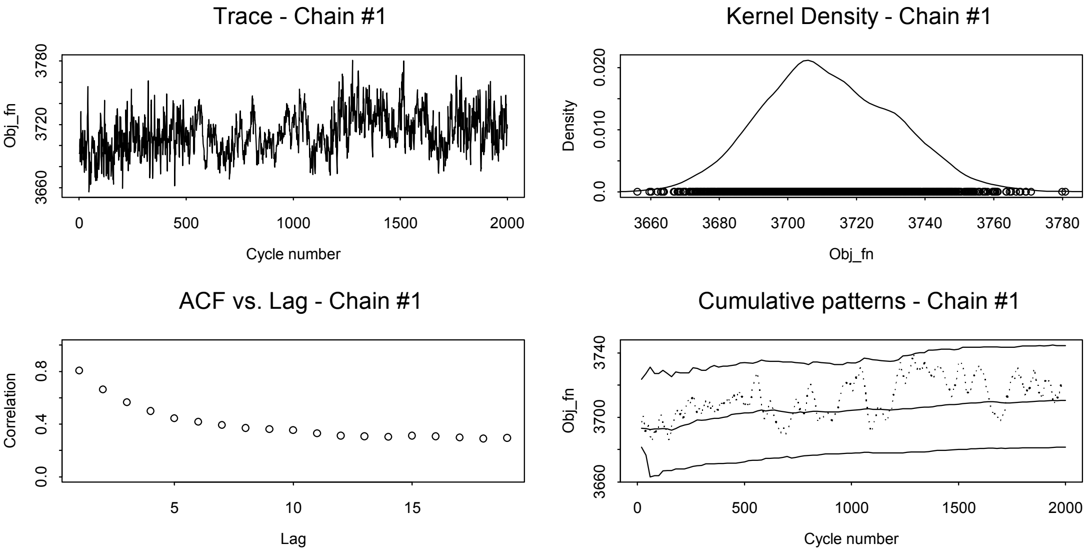


Figure 4.20. Diagnostic statistics for the objective function based on the samples from the MCMC algorithm.

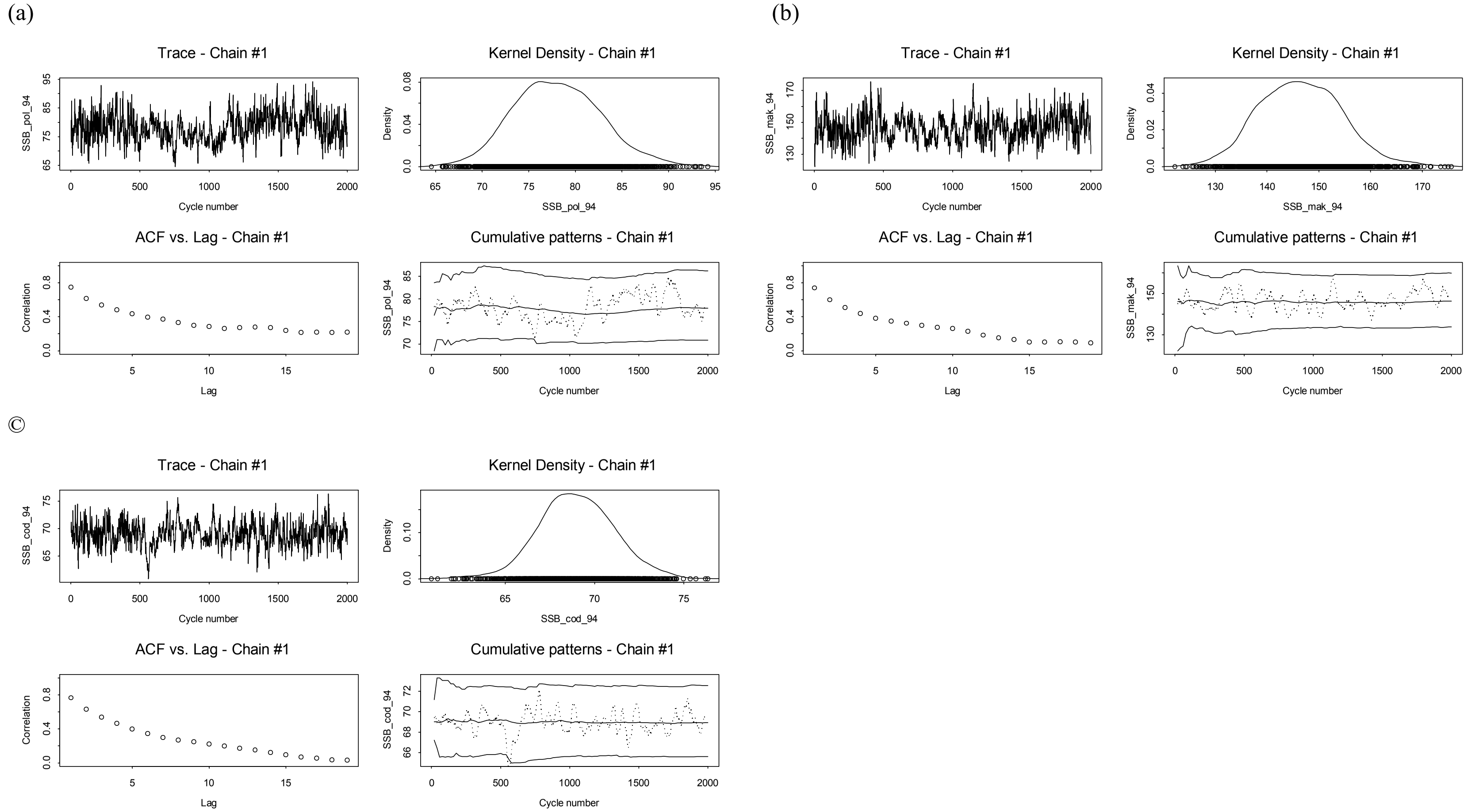


Figure 4.21. Diagnostic statistics for the 1994 spawning biomass for (a) walleye pollock, (b) Atka mackerel and (c) Pacific cod based on the samples from the MCMC algorithm.

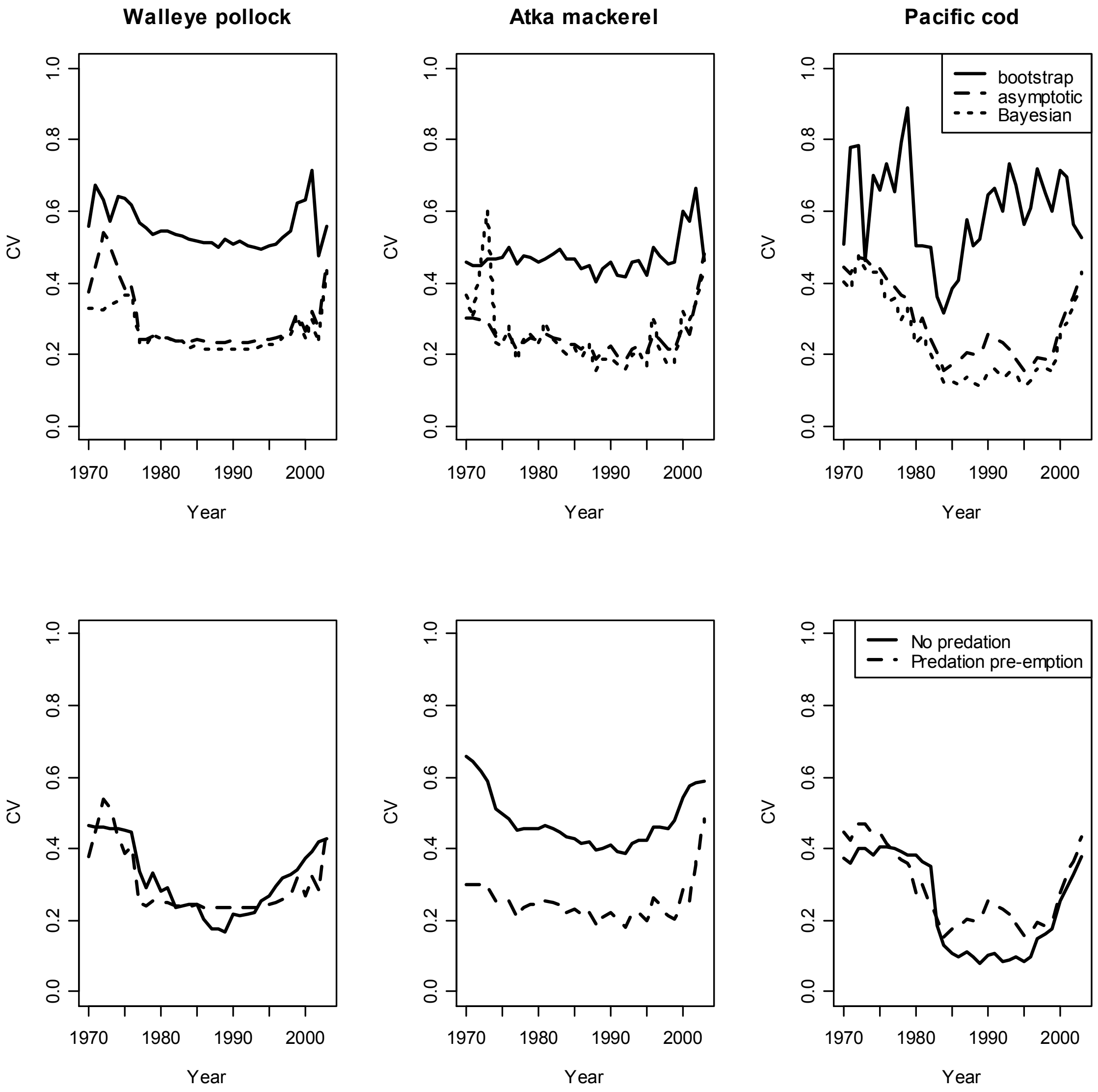


Figure 4.22. Time-trajectories of the coefficient of variation of recruitment (age-0 abundance) for the model based on the predator pre-emption feeding functional relationship using three alternative methods of quantifying uncertainty (upper panels) and time-trajectories of the (asymptotic) coefficient of variation of recruitment based on the "no predation" and base-case "with predation" methods.

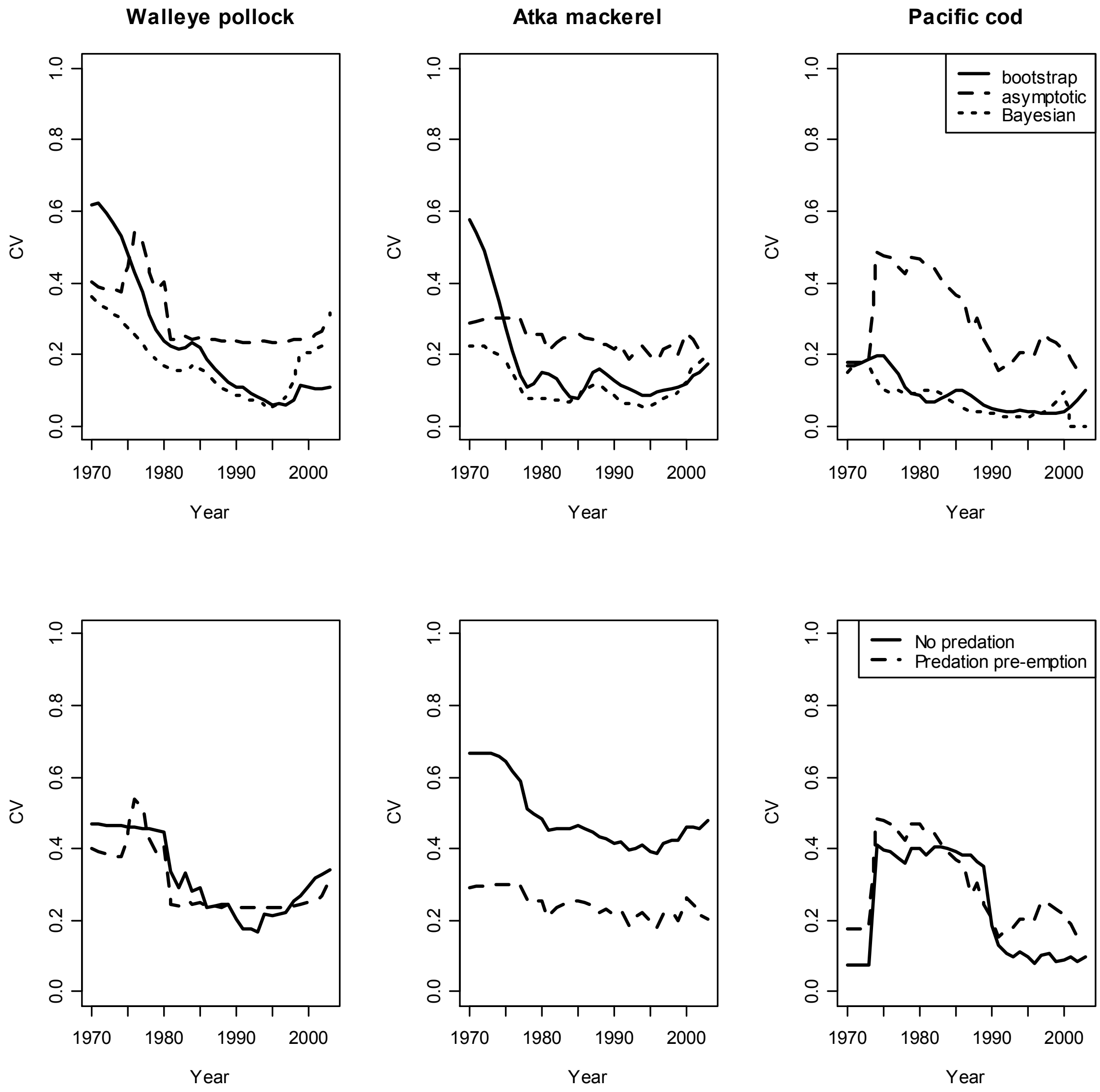


Figure 4.23. Time-trajectories of the coefficient of variation of spawning biomass for the model based on the predator pre-emption feeding functional relationship using three alternative methods of quantifying uncertainty (upper panels) and time-trajectories of the (asymptotic) coefficient of variation of spawning biomass based on the "no predation" and base-case "with predation" methods.

5. SIMULATION EVALUATON

The standard way to assess whether a new statistical estimation method can perform better than existing methods is Monte Carlo simulation. Monte Carlo simulation is therefore used to address the following questions:

- Does a multi-species assessment model provide more reliable predictions of recruitment (age-0 abundance) and spawning biomass than equivalent models that ignore multi-species interactions?
- Is it possible to use diet data to distinguish reliably among alternative feeding functional relationships?
- How does the accuracy and precision of model outputs from multi-species models pertain to the sample sizes for diet composition used for model fitting purposes, e.g. what is the relationship between sample size and the bias / precision of the estimates of the parameters of the feeding functional relationship.

5.1 Methods

The steps when using Monte Carlo simulation to evaluate alternative estimation methods (e.g. Hilborn, 1979; Mapstone *et al*., 1996; Punt *et al*., 2002; Fig. 5.1) are:

1. Specification of a model of nature (the operating model); this model defines the truth for the simulations. For the purposes of the analyses of this chapter, this model is the multi-species model defined in Chapter 3 where the values for its parameters are set by fitting it to the actual data for Pacific cod, Atka mackerel and walleye pollock under two different assumptions about the true form of the feeding functional relationship: 1) Type I, and 2) predator pre-emption. These two predation models were chosen because the Type I relationship is the simplest of the models considered (see Section 4.2) while the predator pre-emption model led to the best fits to the actual data for walleye pollock, Atka mackerel, and Pacific cod in the Aleutians (see Table 4.2).
2. Identification of alternative data collection schemes, viz. sample sizes, specifications for the frequency of collection of data, and the range of predator sizes to sample.
3. Generation of alternative data sets based on each of the alternative data collection schemes and analysis of these data using the modeling framework outlined above. Given specifications for the frequency of sampling, the diet weight- and length-composition data are assumed to be multinomial samples about the expected frequencies, while the daily rations are assumed to be log-normally distributed about their expected values. The pseudo survey abundance data are assumed to be normally distributed about their expected values with CVs based on the actual data. The pseudo proportion-at-age data for the surveys and fisheries are assumed to be multinomially distributed with effective sample sizes as assumed when fitting the model to the actual data.
4. Application of each candidate estimator to each pseudo data set (50 data sets per scenario[4]). For the purposes of this study, the alternative estimators are: a) a

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[4] A number sufficiently large to be able to evaluate qualitative differences among alternative model formulations and data set choices.

model that ignores predation and b) models that include predation based on each of the different feeding functional relationships.

5. Quantification of the results of the simulations in terms of the bias and precision of the estimates of key model outputs, whether model selection criteria correctly identify the true feeding functional relationship, and the relationship between bias and precision and sample size.

5.1.1 Experimental treatments

5.1.1.1 Predation versus non-predation models

A simulation experiment is used to evaluate whether allowing for multi-species effects leads to more reliable (less biased and more precise) estimates of recruitment (age-0 abundance) and spawning biomass. This experiment involves applying three estimation methods to the pseudo data sets generated based on the Type I and predator pre-emption feeding functional relationships (the two operating models). The estimation methods are 1) the model that ignores predation; 2) the simplest variant, i.e. assuming a Type I feeding functional relationship; and 3) the model selected for walleye pollock, Atka mackerel and Pacific cod in the Aleutians, i.e. a predator pre-emption feeding functional relationship. This experiment therefore examines the implications of using predation versus non-predation models in terms of the bias and precision of key model outputs when the feeding functional relationship is known exactly, as well as the implications of incorrect assumptions regarding the feeding functional relationship when conducting multi-species assessments using MLMAK. The data sets for this experiment are based on the survey standard deviations and effective sample sizes on which the analyses in Chapter 4 were based.

5.1.1.2 Selecting the correct feeding functional relationship

This experiment involves generating data from models in which predation is governed either by the Type I or the predator pre-emption feeding functional relationships and then applying models based on six of the seven feeding functional relationships[5]. AIC is then used to select among the seven feeding functional relationships. The objective of this experiment is to determine how often the correct feeding functional relationship is selected.

5.1.1.3 Impact of increased sample sizes

The stomach sample sizes for the actual data are relatively low (see Tables A.12, A.14, A.16, A.18, A.20 and A.22). The previous two experiments are therefore repeated: (a) when the sample sizes for the diet data are quadrupled, and (b) when the sample sizes are set to 100 for all sizes classes and years (rather than just the years for which data were actually collected).

5.2 Results and discussion

5.2.1 Predation versus non-predation models

Figures 5.2-5.4 show the relative error distributions for recruitment (age-0 abundance), recruitment expressed relative to the mean recruitment over the period considered in the model, and spawning biomass for the three estimation methods when the operating model

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[5] Results are not shown for analyses based on the Type III feeding functional relationship owing to convergence problems.

is a model that includes predation with a Type I feeding functional relationship. Results are shown in Figure 5.3 for recruitment expressed relative to the mean recruitment to assess how well each estimation model is able to determines trends in (rather than absolute levels of) recruitment.

The “no predation” model performs poorly in terms of estimating recruitment in absolute terms for walleye pollock (Fig. 5.2), although this is perhaps not surprising because walleye pollock is preyed on by both Atka mackerel and Pacific cod, and predation mortality on young pollock is consequently estimated to be very high. The estimation methods that account for predation lead to less biased estimates of recruitment of walleye pollock and Atka mackerel, although the estimates are less precise than those obtained using the “no predation” model. In contrast to the situation for walleye pollock and Atka mackerel, all three methods lead to estimates of cod recruitment with similar levels of bias and precision (Fig. 5.2). When expressed relative to mean recruitment, the “with predation” estimation methods again outperform the “no predation” estimation method for walleye pollock (Fig. 5.3), but the size of the effect is much less than was the case in Figure 5.2. The estimation method based on a Type I feeding functional relationship leads to estimates of recruitment relative to mean recruitment with the lowest bias and variance (Fig. 5.3), although this is perhaps not unexpected given that the operating model for these simulations is based on a Type I feeding functional relationship. The median absolute relative error (MARE) is a statistic that combines the impact of bias and precision, but is less sensitive to outliers than the root mean square error. This statistic has been used in several previous studies of estimator performance (e.g. Punt, 2003; Wilberg and Bence, 2006). Figure 5.5 contrasts the ability of the three estimators to estimate recruitment and spawning biomass using this measure. Figure 5.5 confirms that the “with predation” models lead to better estimates of recruitment of walleye pollock than the “no predation” model, while the situation is unclear for mackerel and there is little to choose between the three estimators for Pacific cod.

The “with predation” estimation methods lead to estimates of spawning biomass that are closer to being unbiased than the “no predation” estimation method, but these estimates are less precise (Fig. 5.4). When expressed as median absolute relative errors (Fig. 5.5), estimation based on a predator pre-emption feeding functional relationship is best for Atka mackerel. The “no predation” model leads to estimates with the lowest MAREs for Pacific cod for the years 1985-90, but there is otherwise little to choose among the three estimation methods for Pacific cod. The situation for walleye pollock is complicated with the estimation method based on the predator pre-emption feeding functional relationship leading to high MAREs for the early years (presumably because of the poor precision – Fig. 5.4c), the “no predation” method being best between 1975 and 1980 and after 2002, and the estimation method based on a Type I feeding functional relationship leading to the lowest MAREs for the remaining years.

Figures 5.2-5.5 were based on an operating model in which the true feeding functional relationship was a Type I relationship. Figures 5.6-5.9 shows the same results as Figures 5.2-5.5, but for the case in which the operating model involves the predator pre-emption feeding functional relationship. The estimation method based on the predator pre-emption model performs better than the other two methods fairly consistently in contrast to the case when the operating model was based on the Type I feeding functional relationship when the estimation method based on a Type I feeding functional relationship was only marginally better than the other two methods (if it was better at all). This is probably caused by the Type I feeding functional relationship being nested within the predator pre-emption feeding functional relationship. The performance of all of the estimation methods is poor in terms of estimating spawning biomass for the earliest years of the modelled period, even when the correct feeding functional relationship is assumed (Fig. 5.8c).

5.2.2 Selecting the correct feeding functional relationship

Figure 5.10 summarizes the results of the experiment in which six "with predation" models are applied to data sets generated by an operating model with: a) a Type I feeding functional relationship, and b) a predator pre-emption feeding functional relationship, in terms of the percentage of times each alternative feeding functional relationship is selected using AIC. As was the case when assessing the ability to estimate recruitment and spawning biomass, there are noteworthy differences between the two operating models. Specifically, the probability of selecting the correct feeding functional relationship is low when the Type I feeding functional relationship is correct, but high for the case in which the predator pre-emption feeding functional relationship is correct. This could again be attributed to the fact that the Type I feeding functional relationship is nested within the other five feeding functional relationships.

5.2.3 Impact of increased sample sizes

Figures 5.11 and 5.12 contrast the ability of the three estimators to estimate recruitment and spawning biomass based on the Median Absolute Relative Error when the diet weight and length sample sizes are quadrupled while Figures 5.13 and 5.14 show these MAREs when there are diet data for all years from 1980 and the sample size for each combination of predator species and predator length-class is 100. As expected, increasing the diet composition sample sizes generally leads to more accurate and precise estimates (see Figures 5.15 and 5.16 which contrast the MAREs for each alternative data collection scheme for the case in which the estimation and operating models are identical). The effect of increased diet composition sample sizes is more marked for pollock and mackerel when the operating model is the Type I feeding functional relationship (Figure 5.15) [results not shown here show that the MAREs for the predator pre-emption feeding functional relationship exhibit the same patterns as those in Figure 5.15 when the operating model is based on the Type I feeding functional relationship].

The results for the predator pre-emption feeding functional relationship (Fig. 5.16) are more complicated than those for the Type I feeding functional relationship. The estimates of age-0 abundance for mackerel and (particularly) pollock are better with the largest data set, but the estimates of age-0 abundance for mackerel are very poor for the earliest years, which impacts the time-trajectories of spawning biomass for this species.

Figure 5.17 suggests that increasing the diet composition sample sizes does not markedly increase the probability of selecting the Type I feeding functional relationship when this feeding functional relationship is correct. In fact, the second simplest feeding functional relationship is selected most frequently using AIC when the Type I feeding functional relationship is correct and diet composition samples are available for all predator species and length-classes since 1980.

5.2.4 General discussion

The results of this chapter indicate that including predation in stock assessment models and fitting such models to diet composition data will not necessarily lead to improved estimates of age-0 abundance and spawning biomass, and that any such improvements are likely to depend on the quality of the diet composition data. Most management regimes for target species depend on estimates of spawning biomass and the abundance of the population vulnerable to the gear (e.g. Anon. 2005). The results of this chapter suggest that relatively little is to be gained by using estimation methods that include predation if the aim is to estimate these quantities. In contrast, the estimates of age-0 abundance (in absolute terms) can be improved markedly by using a predation model. Estimates of age-0 abundance in absolute terms could be useful for assessing the food available to predator species (such as Steller Sea Lions in the case of pollock, mackerel and cod in the Aleutians).

The analyses suggest that the ability to detect the correct feeding functional relationship using data on diet composition and abundance may be relatively weak (Figs 5.2 and 5.17). However, it is perhaps noteworthy that it was possible to correctly detect a complicated feeding functional relationship, but not one that was nested within a suite of alternative feeding functional relationships. Moreover, the ability to estimate quantities of management interest such as spawning biomass and age-0 abundance was not markedly impacted by an inability to correctly detect the correct form of the feeding functional relationship (within the range of feeding functional relationships that are able to mimic the data adequately).

The analyses of this chapter only considered modifications to the sample sizes for the diet composition data. However, the performance of the estimation methods also depend on the sample sizes for the other data types included in the assessment, specifically the survey estimates of abundance and the catch and survey age- /length-composition data. Results not shown here indicate that markedly increasing the sample sizes for these data types as well as those for the diet composition data leads to estimates of age-0 abundance and spawning biomass with very small median absolute relative errors.

One potentially major weakness of all of the models considered in this chapter (estimation and operating) is that they all have annual time-steps and are fitted to information on diet composition collected during summer surveys. Unfortunately, there are no data to estimate diet compositions during other seasons and this could lead to poor estimation performance, but there is at present no way to quantify the extent of bias. Collection of additional diet composition data could help to resolve this source of uncertainty.

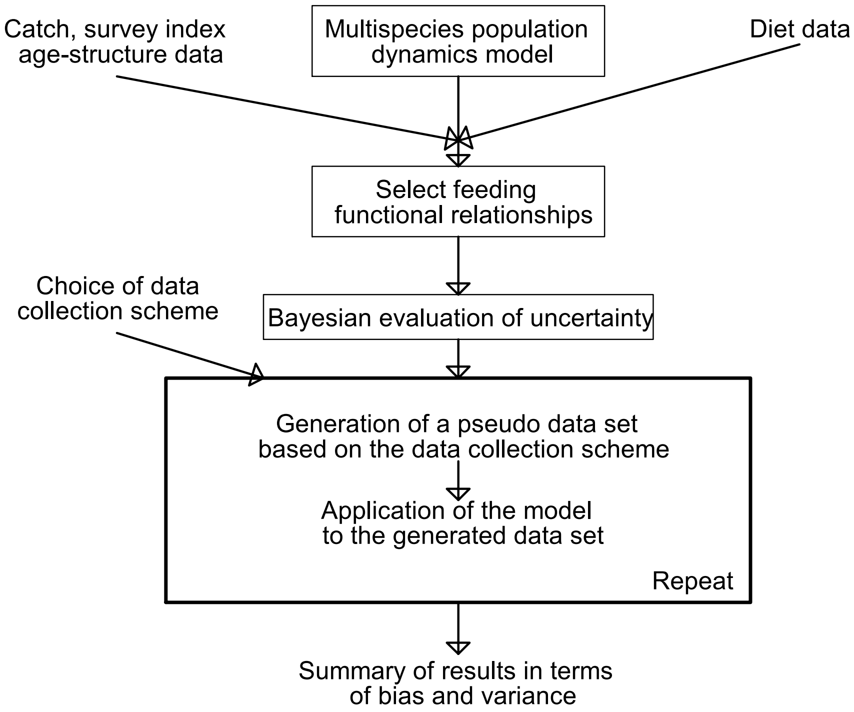
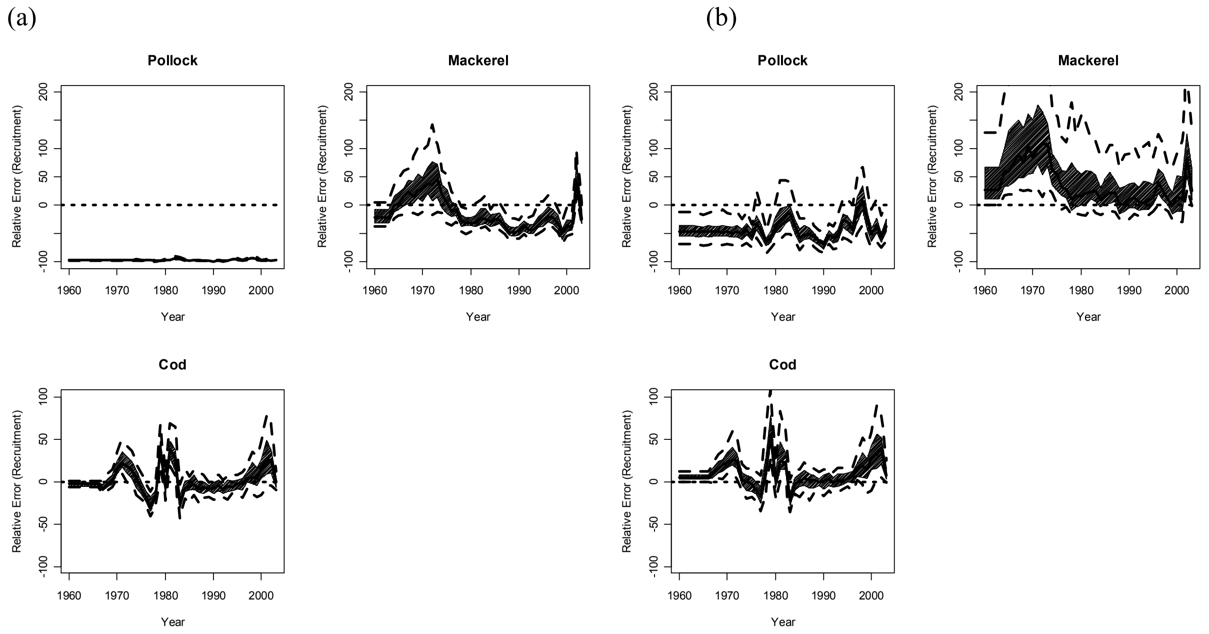


Figure 5.1. Summary of the process of using simulation to evaluate a data collection scheme.



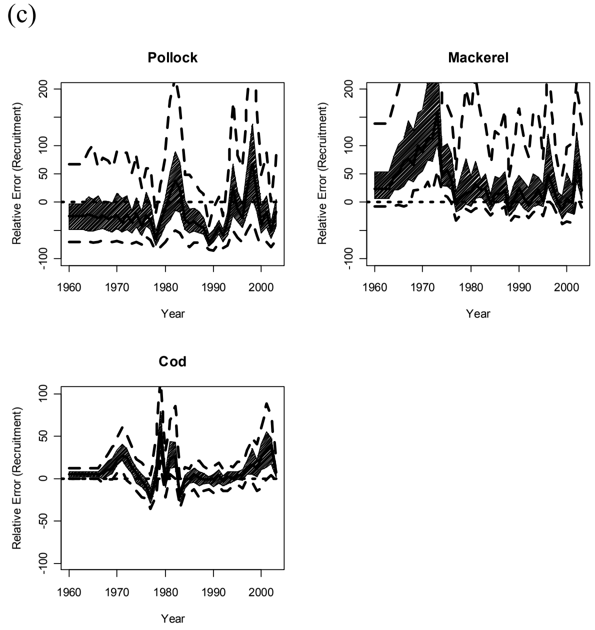


Figure 5.2. Time-trajectories of relative error for recruitment. Results are shown for (a) the "no predation" model, (b) the predation model with a Type I feeding functional relationship, and (c) the predation model with a predator pre-emption feeding functional relationship. The operating model is the predation model with a Type I feeding functional relationship.

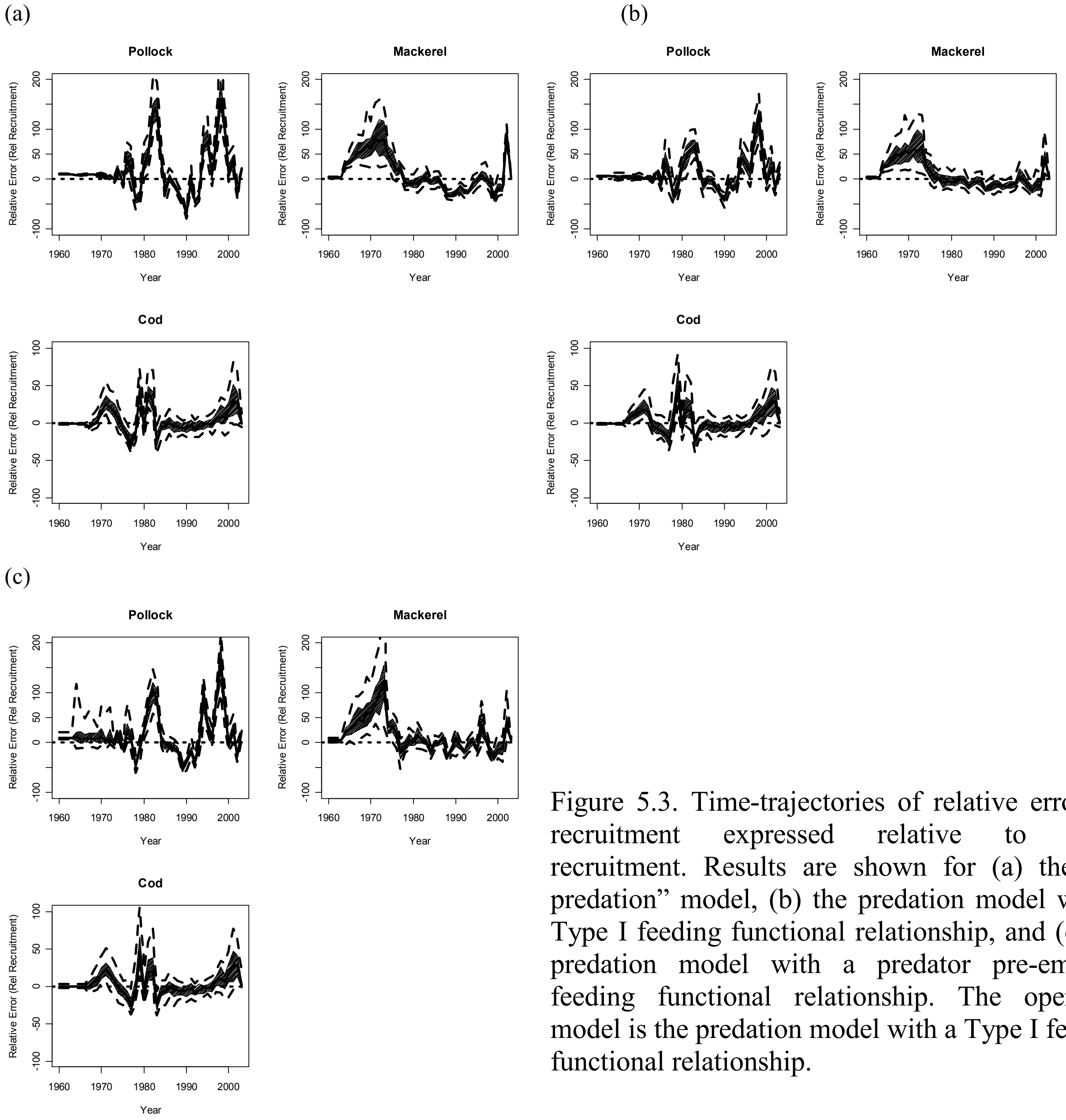
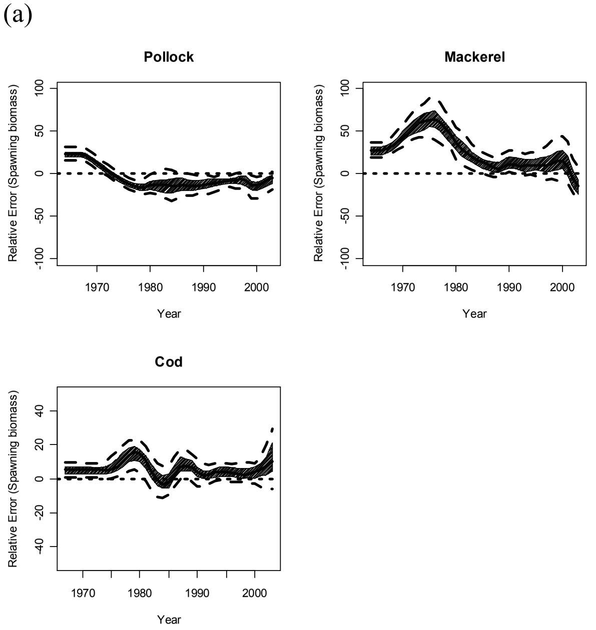
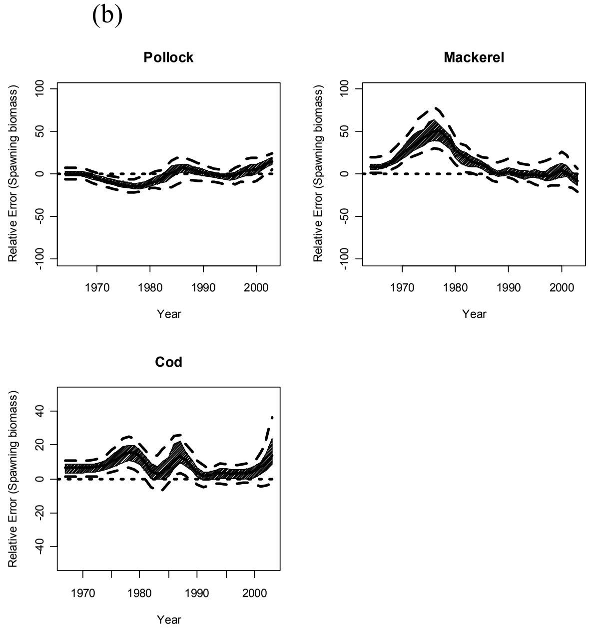


Figure 5.3. Time-trajectories of relative error for recruitment expressed relative to mean recruitment. Results are shown for (a) the “no predation” model, (b) the predation model with a Type I feeding functional relationship, and (c) the predation model with a predator pre-emption feeding functional relationship. The operating model is the predation model with a Type I feeding functional relationship.





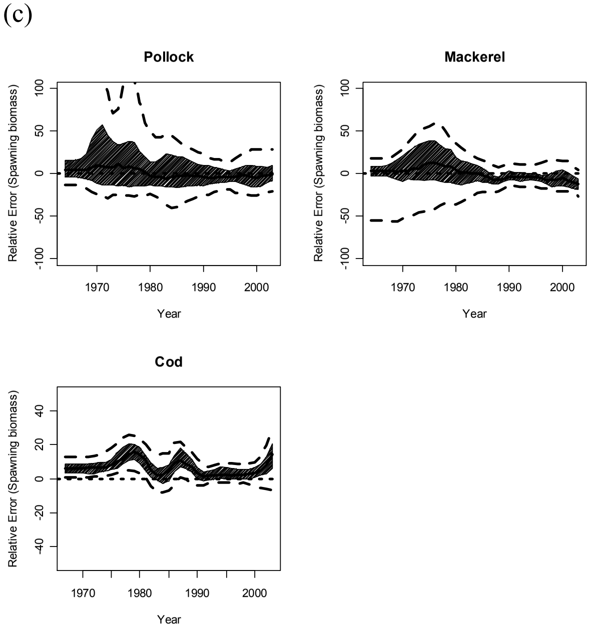


Figure 5.4. Time-trajectories of relative error for spawning biomass. Results are shown for (a) the "no predation" model, (b) the predation model with a Type I feeding functional relationship, and (c) the predation model with a predator pre-emption feeding functional relationship. The operating model is the predation model with a Type I feeding functional relationship.

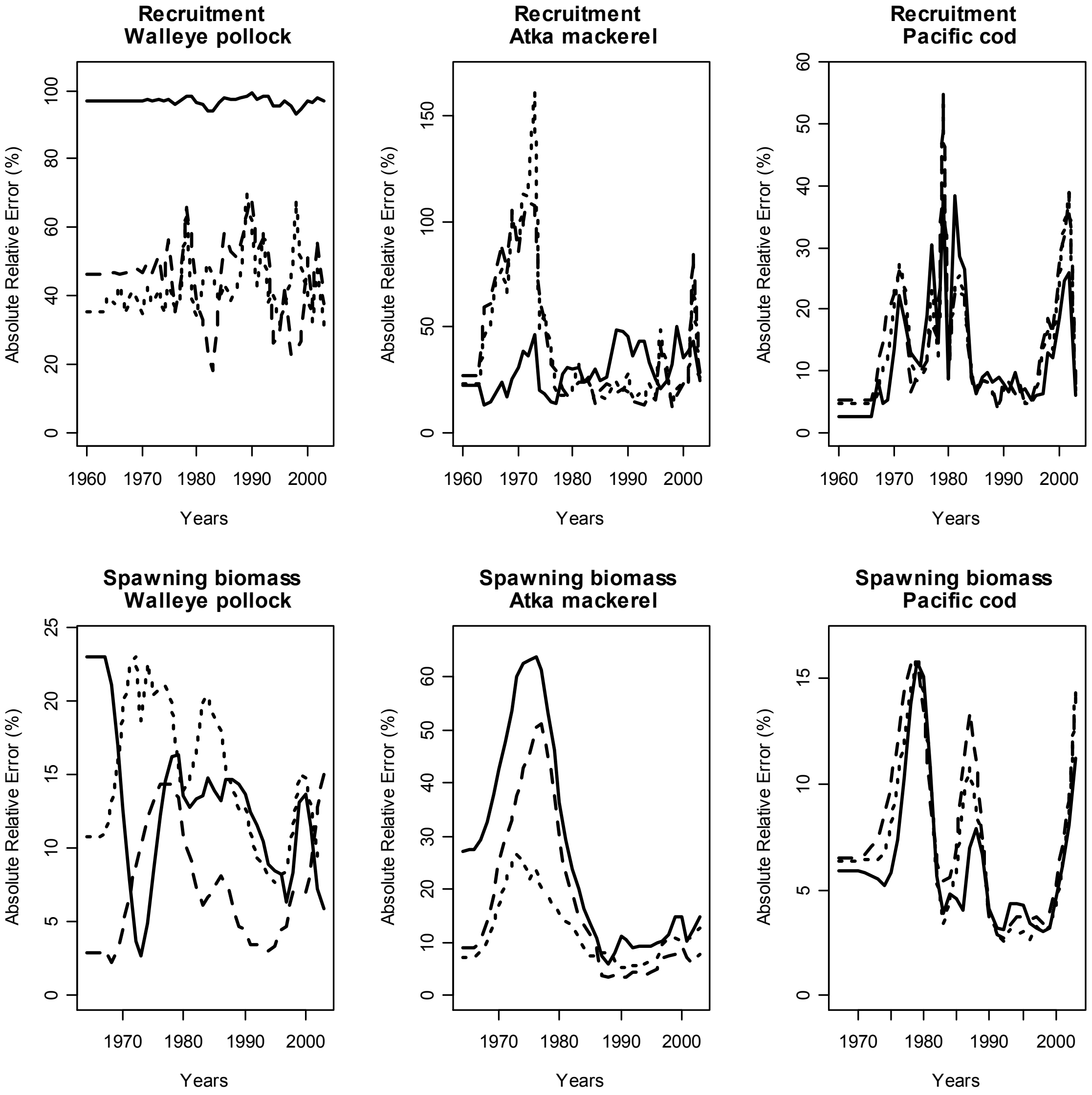
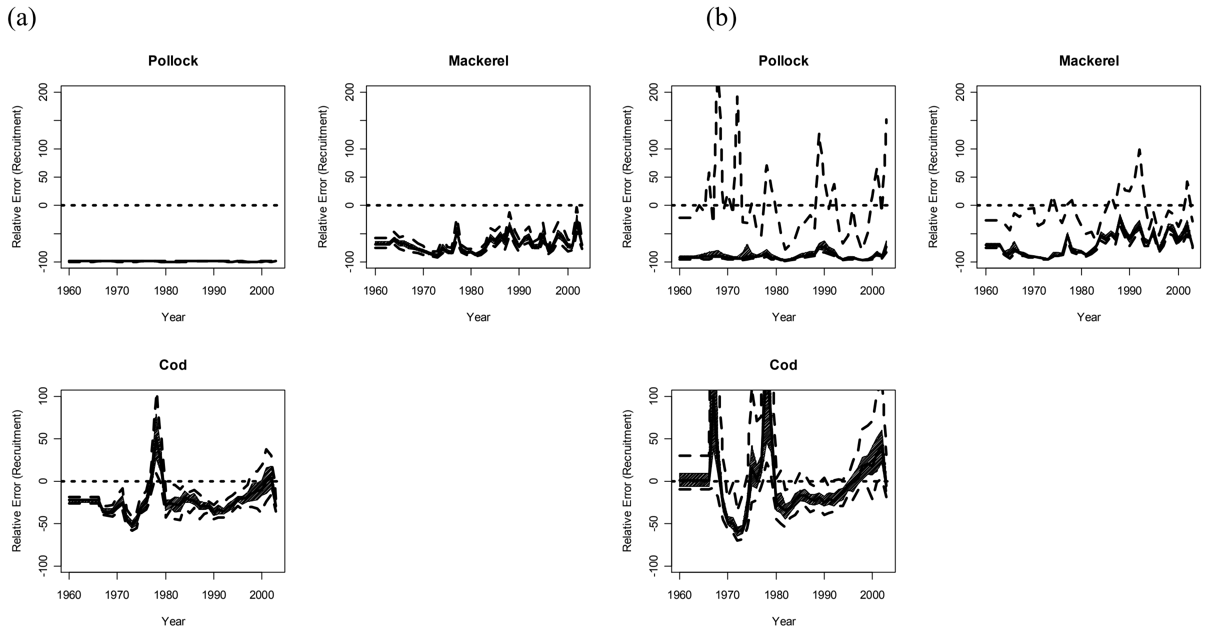


Figure 5.5. Time-trajectories of median absolute relative error for recruitment (upper panels) and spawning biomass (lower panels). Results are shown for (a) the "no predation" model (solid lines), (b) the predation model with a Type I feeding functional relationship (dashed lines), and (c) the predation model with a predator pre-emption feeding functional relationship (dotted lines). The operating model is the predation model with a Type I feeding functional relationship.



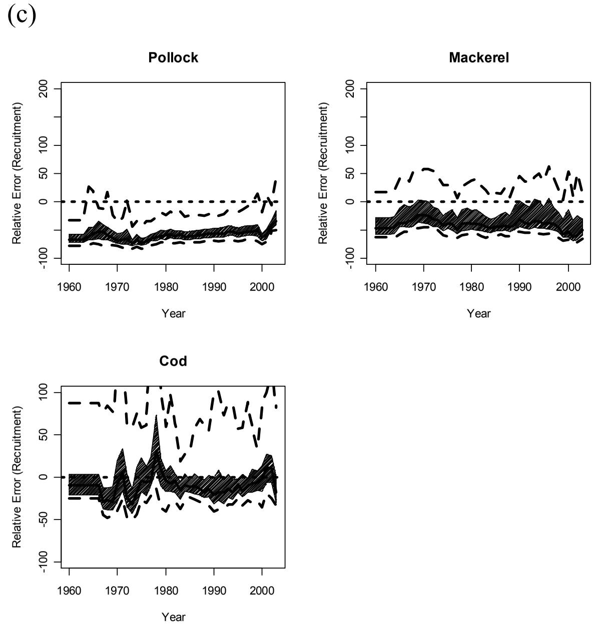


Figure 5.6. Time-trajectories of relative error for recruitment. Results are shown for (a) the "no predation" model, (b) the predation model with a Type I feeding functional relationship, and (c) the predation model with a predator pre-emption feeding functional relationship. The operating model is the predation model with a predator pre-emption feeding functional relationship.

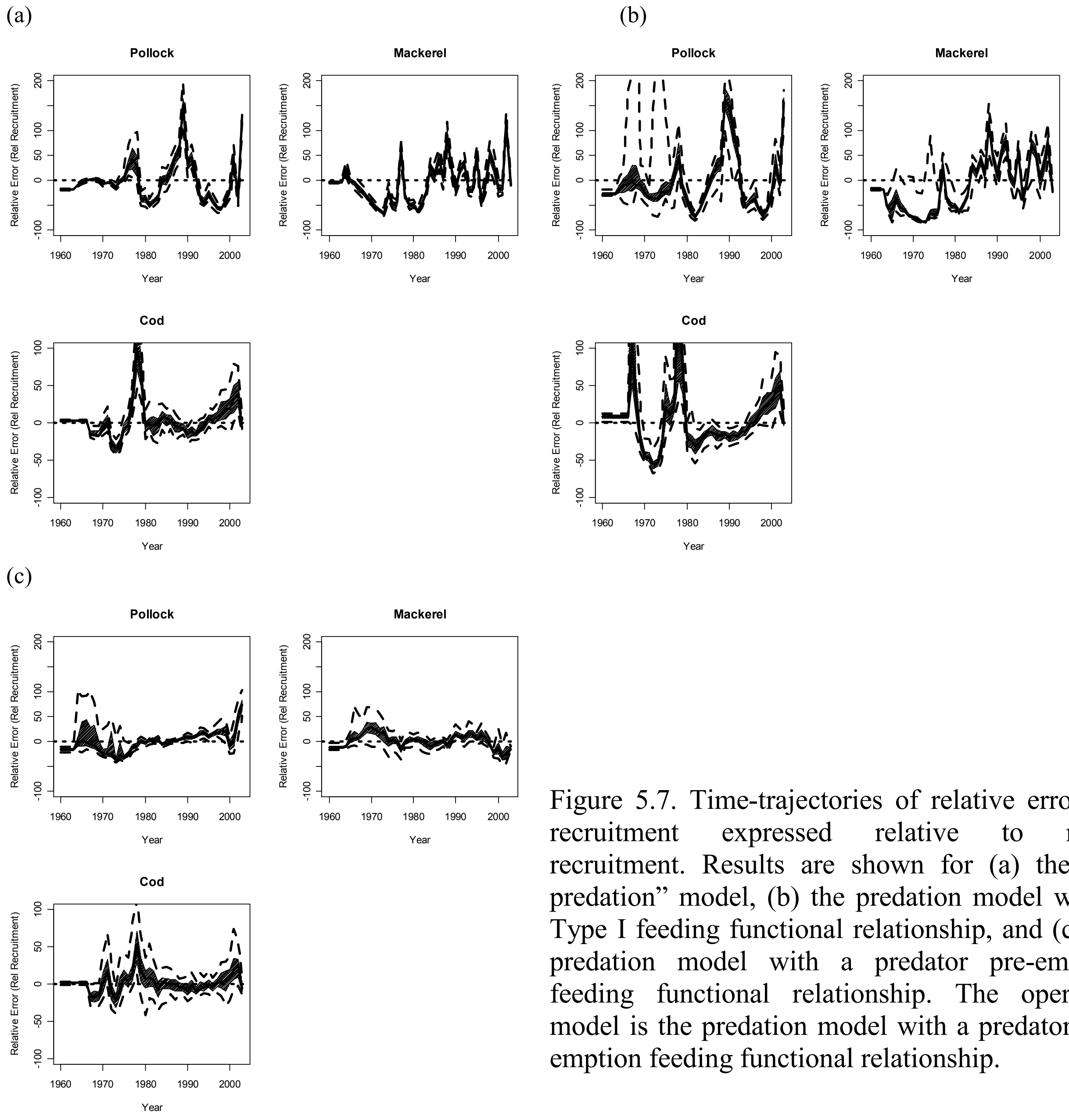


Figure 5.7. Time-trajectories of relative error for recruitment expressed relative to mean recruitment. Results are shown for (a) the "no predation" model, (b) the predation model with a Type I feeding functional relationship, and (c) the predation model with a predator pre-emption feeding functional relationship. The operating model is the predation model with a predator pre-emption feeding functional relationship.

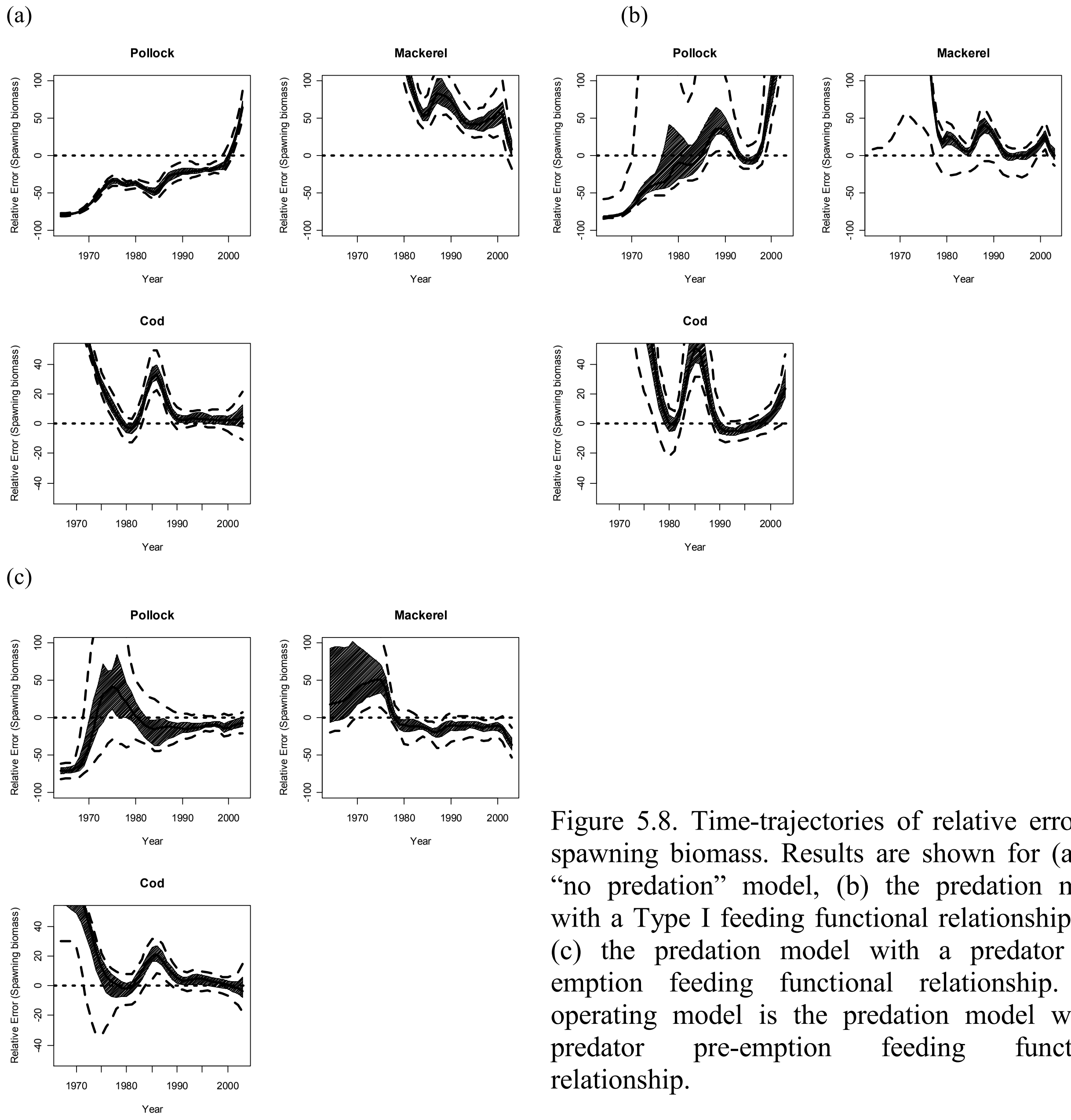


Figure 5.8. Time-trajectories of relative error for spawning biomass. Results are shown for (a) the "no predation" model, (b) the predation model with a Type I feeding functional relationship, and (c) the predation model with a predator pre-emption feeding functional relationship. The operating model is the predation model with a predator pre-emption feeding functional relationship.

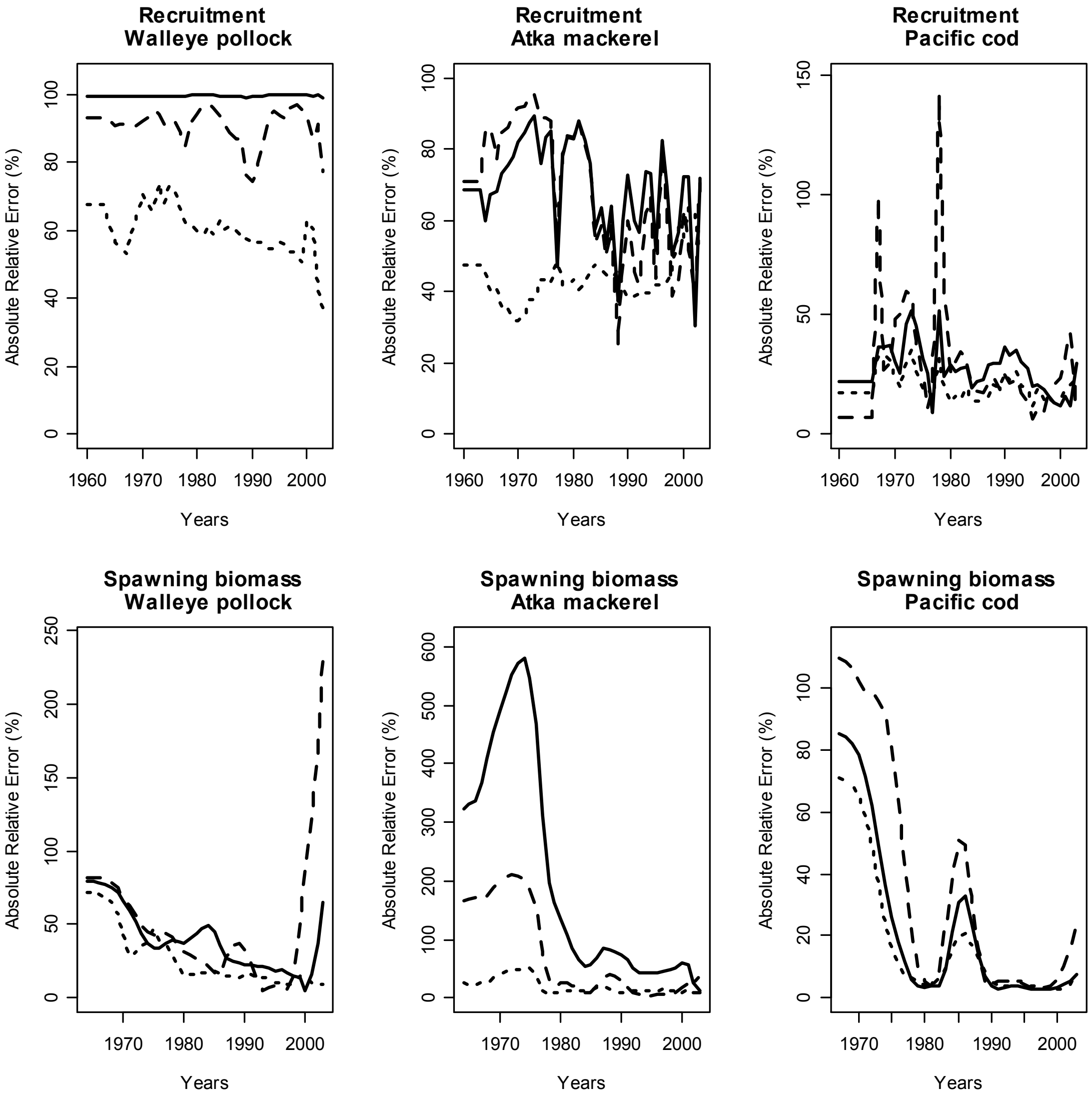
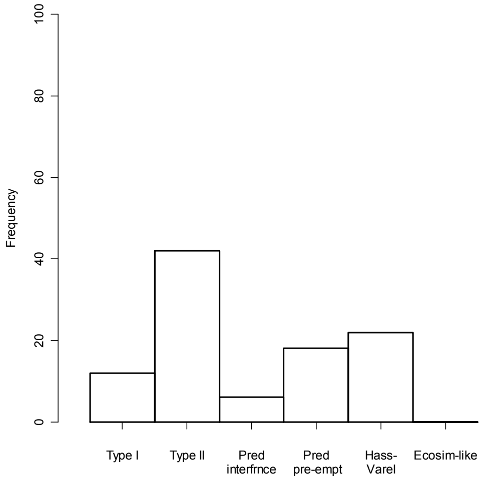


Figure 5.9. Time-trajectories of median absolute relative error for recruitment (upper panels) and spawning biomass (lower panels). Results are shown for (a) the "no predation" model (solid lines), (b) the predation model with a Type I feeding functional relationship (dashed lines), and (c) the predation model with a predator pre-emption feeding functional relationship (dotted lines). The operating model is the predation model with a predator pre-emption feeding functional relationship.

(a) Type I feeding functional relationship



(b) Predator pre-emption feeding functional relationship

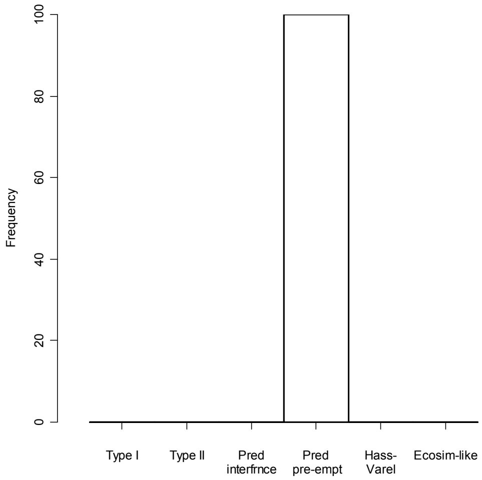


Figure 5.10. Frequency with which each of six alternative feeding functional relationships are selected using AIC for operating models based on (a) a Type I feeding functional relationship, and (b) a predator pre-emption feeding functional relationship.

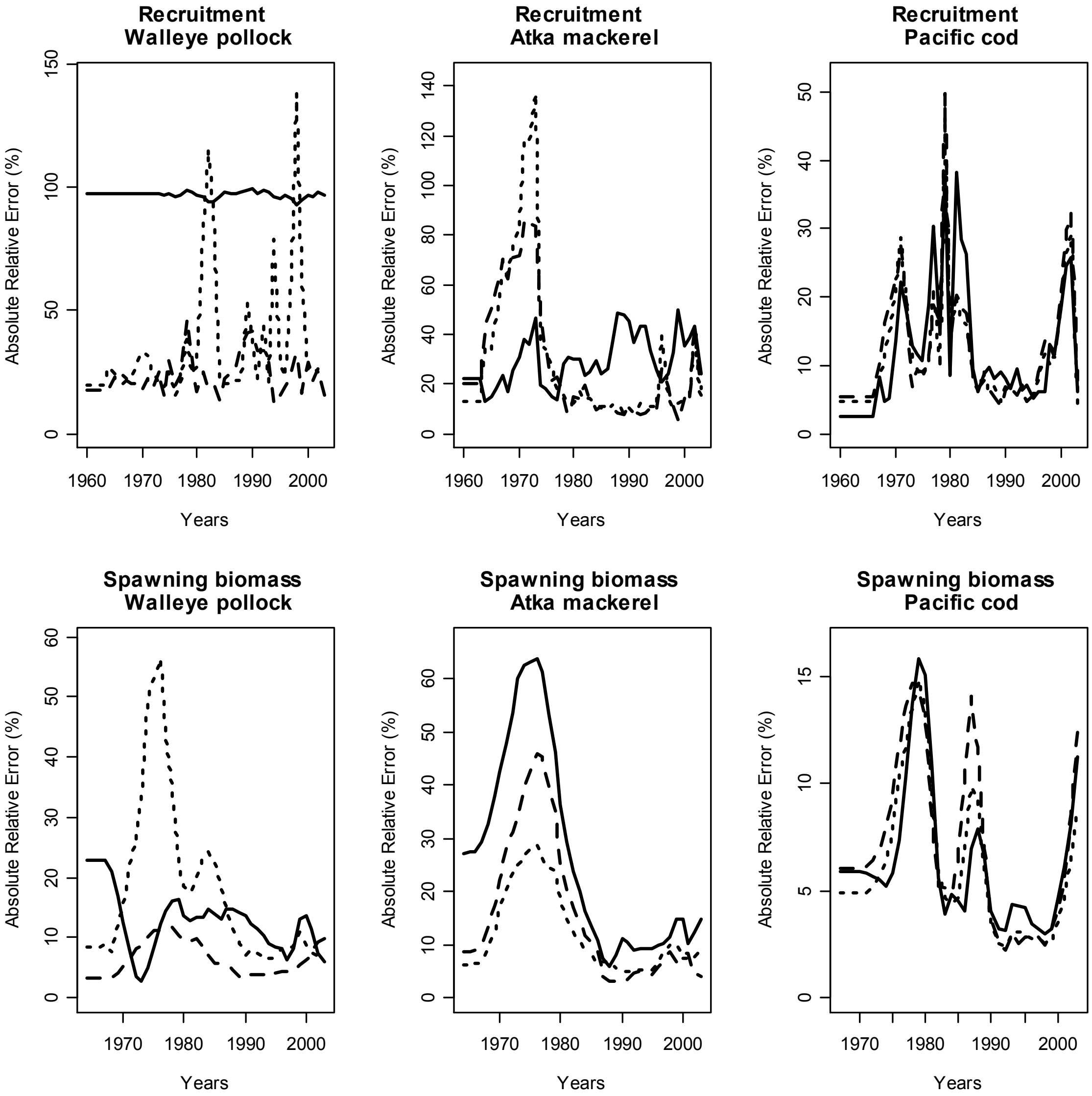
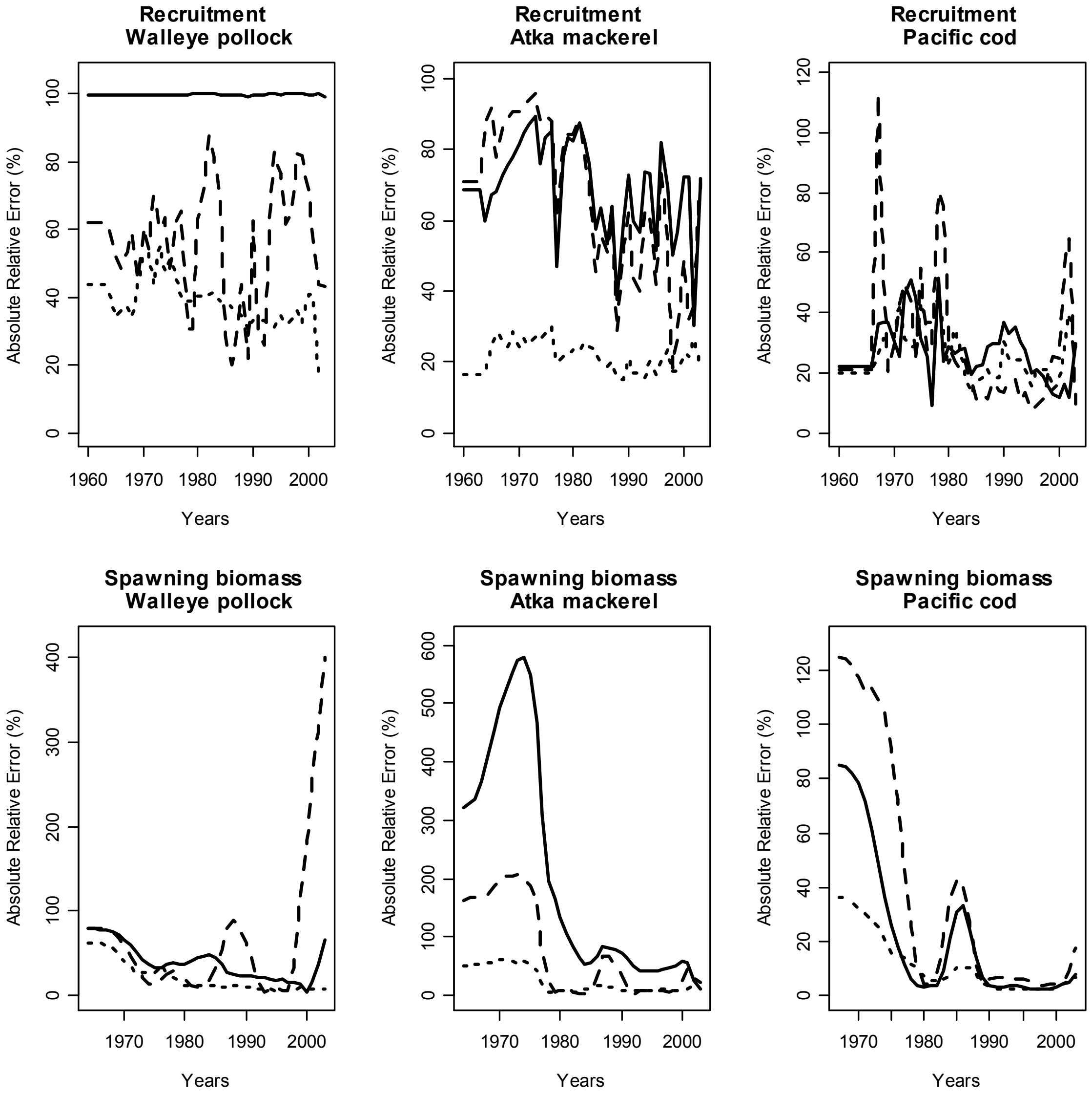


Figure 5.11. Time-trajectories of median absolute relative error for recruitment (upper panels) and spawning biomass (lower panels). Results are shown for (a) the “no predation” model (solid lines), (b) the predation model with a Type I feeding functional relationship (dashed lines), and (c) the predation model with a predator pre-emption feeding functional relationship (dotted lines). The operating model is the predation model with a Type I feeding functional relationship when the diet sample sizes are multiplied by four.



Recruitment Walleye pollock

Recruitment Atka mackerel

Recruitment Pacific cod

Spawning biomass Walleye pollock

Spawning biomass Atka mackerel

Spawning biomass Pacific cod

Figure 5.12. Time-trajectories of median absolute relative error for recruitment (upper panels) and spawning biomass (lower panels). Results are shown for (a) the “no predation” model (solid lines), (b) the predation model with a Type I feeding functional relationship (dashed lines), and (c) the predation model with a predator pre-emption feeding functional relationship (dotted lines). The operating model is the predation model with a predator pre-emption feeding functional relationship when the diet sample sizes are multiplied by four.

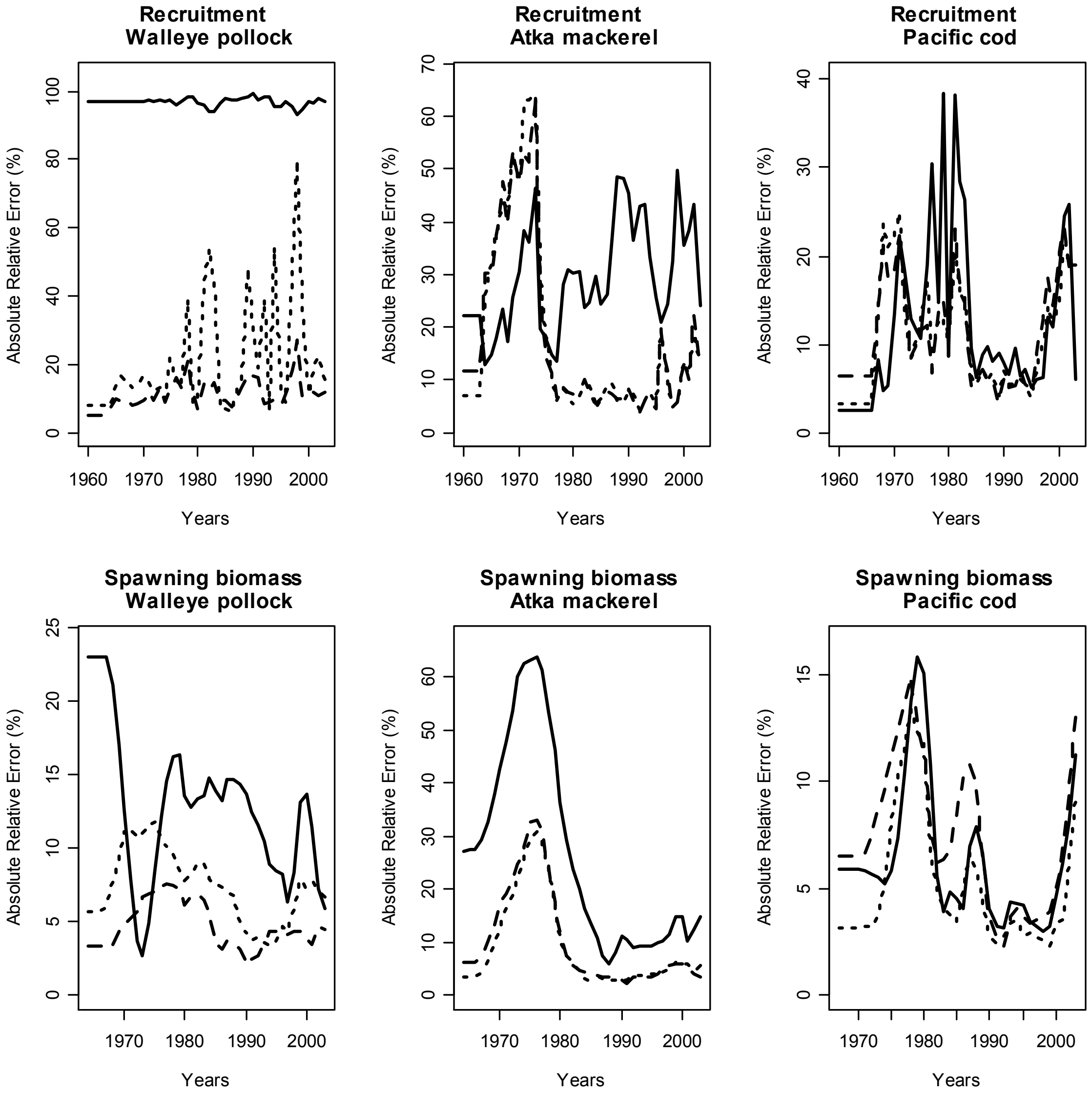


Figure 5.13. Time-trajectories of median absolute relative error for recruitment (upper panels) and spawning biomass (lower panels). Results are shown for (a) the "no predation" model (solid lines), (b) the predation model with a Type I feeding functional relationship (dashed lines), and (c) the predation model with a predator pre-emption feeding functional relationship (dotted lines). The operating model is the predation model with a Type I feeding functional relationship when the diet sample sizes are 100 for years and predator lengths since 1980.

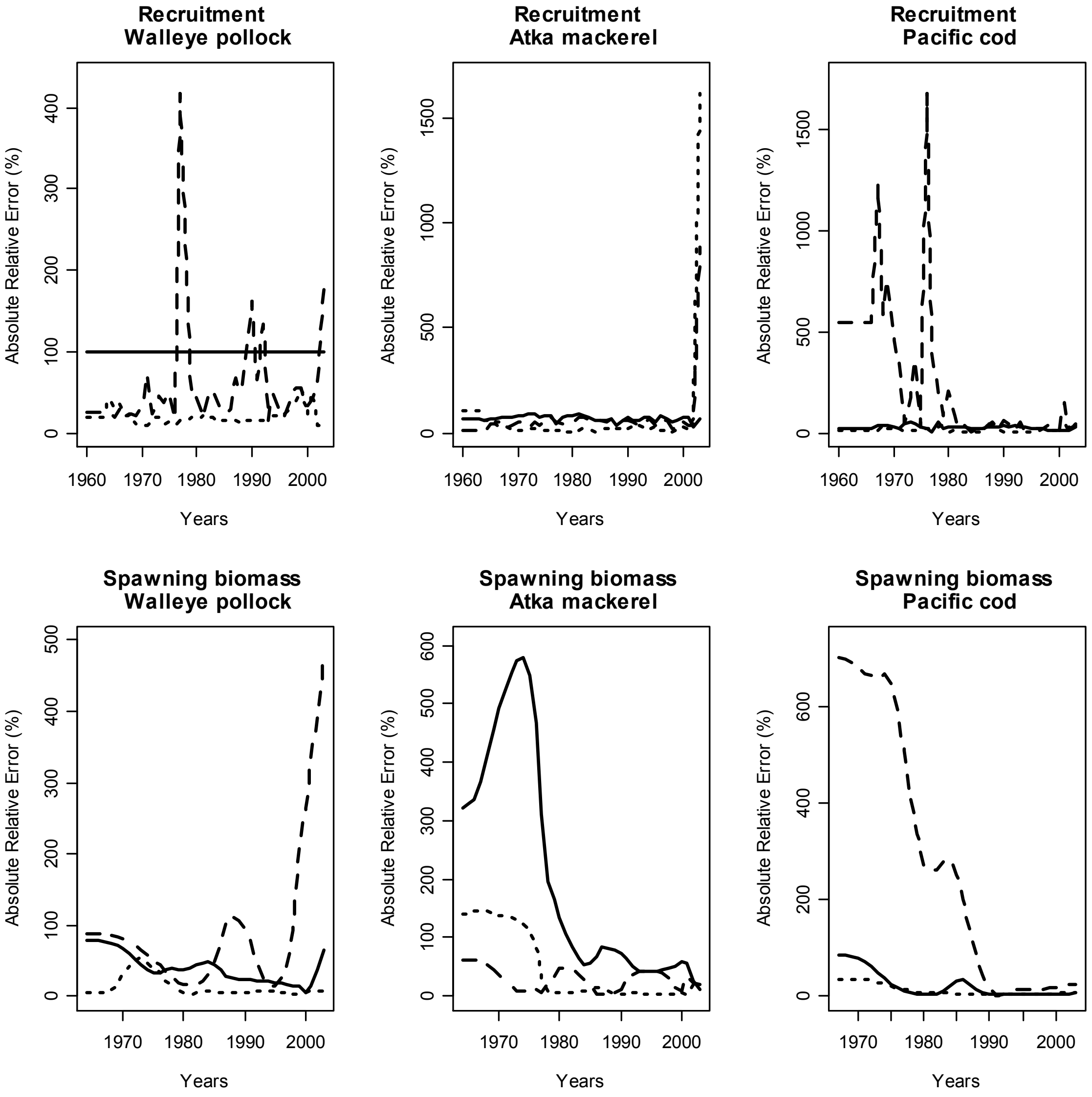


Figure 5.14. Time-trajectories of median absolute relative error for recruitment (upper panels) and spawning biomass (lower panels). Results are shown for (a) the "no predation" model (solid lines), (b) the predation model with a Type I feeding functional relationship (dashed lines), and (c) the predation model with a predator pre-emption feeding functional relationship (dotted lines). The operating model is the predation model with a predator pre-emption feeding functional relationship when the diet sample sizes are 100 for years and predator lengths since 1980.

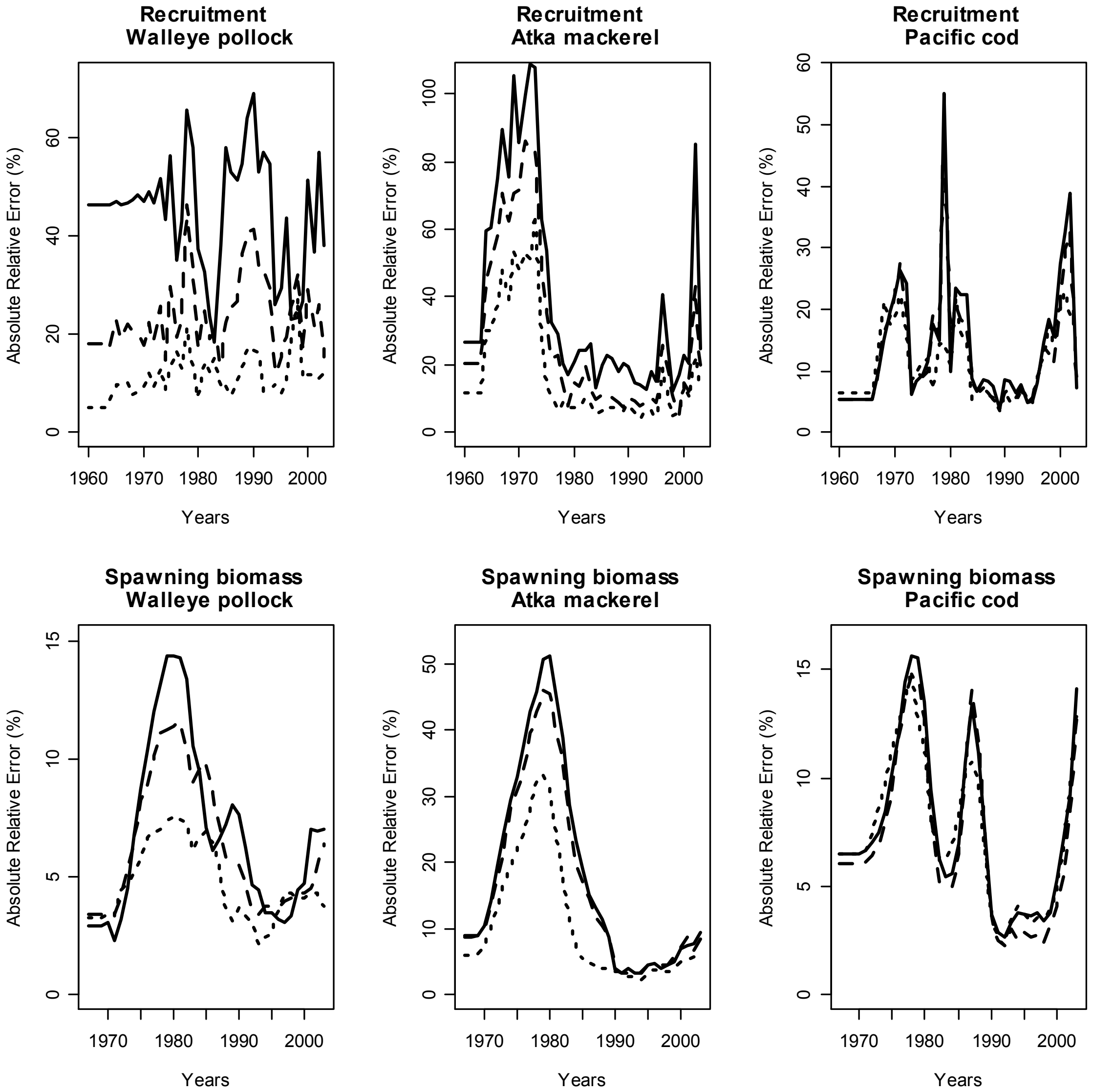


Figure 5.15. Time-trajectories of median absolute relative error for recruitment (upper panels) and spawning biomass (lower panels). Results are shown for (a) the base-case data collection scheme (solid lines), (b) quadrupling the base-case diet sample sizes (dashed lines), and (c) setting the diet sample sizes to 100 for years and predator lengths since 1980 (dotted lines). The operating and estimation model are both the predation model with a Type I feeding functional relationship.

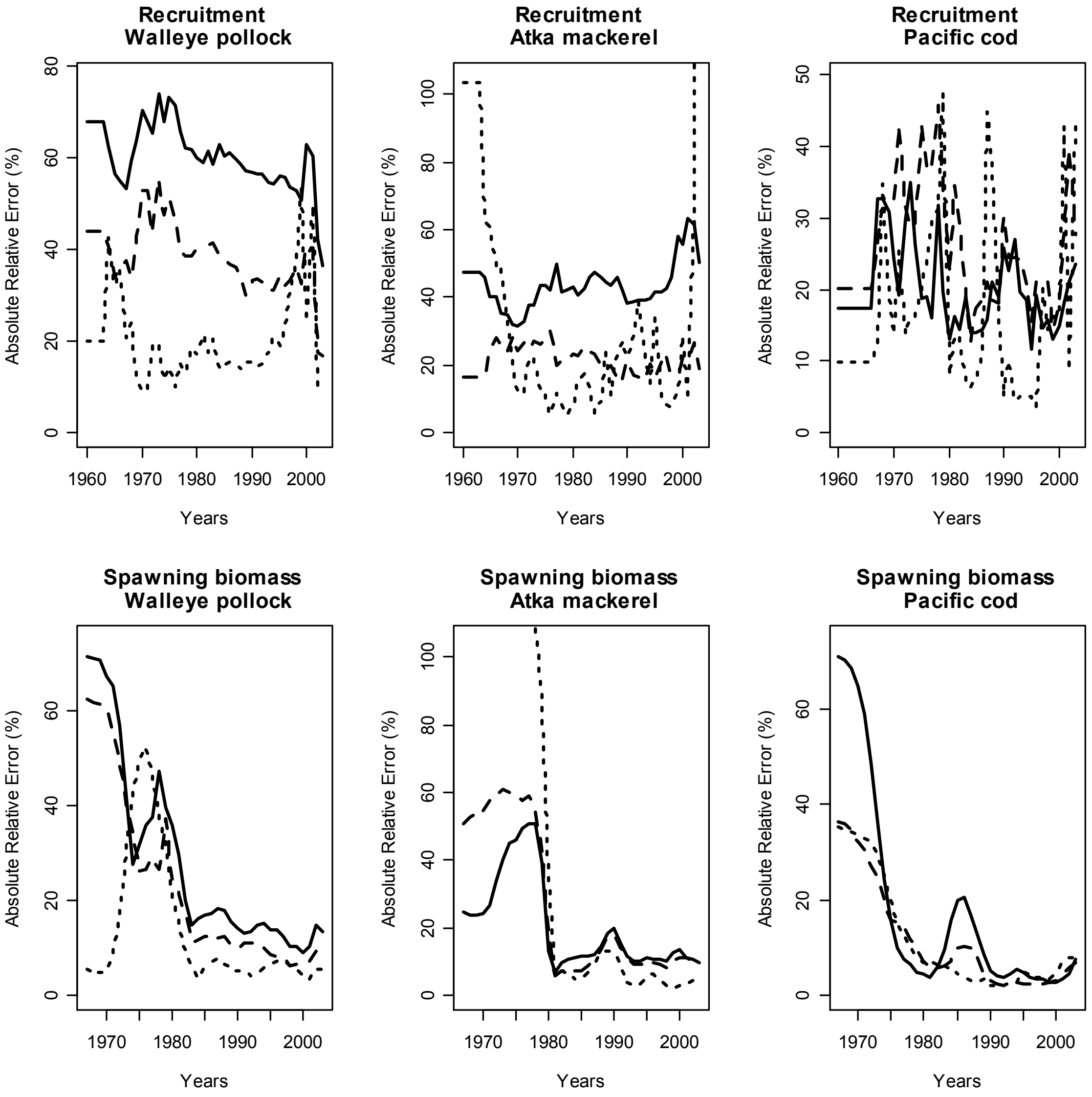
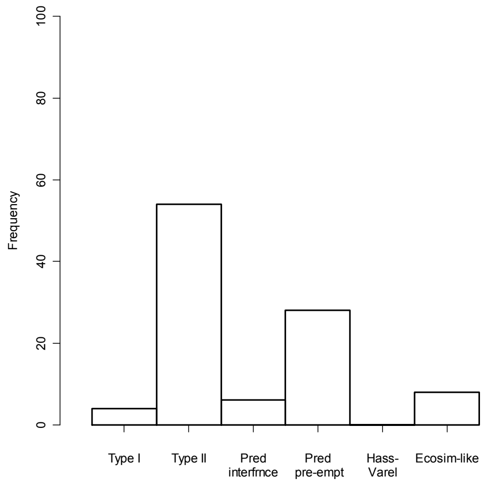


Figure 5.16. Time-trajectories of median absolute relative error for recruitment (upper panels) and spawning biomass (lower panels). Results are shown for (a) the base-case data collection scheme (solid lines), (b) quadrupling the base-case diet sample sizes (dashed lines), and (c) setting the diet sample sizes to 100 for years and predator lengths since 1980 (dotted lines). The operating and estimation model are both the predation model with a predator pre-emption feeding functional relationship.

(a) Diet composition sample size quadrupuled



(b) Diet sample sizes are 100 for years and predator lengths since 1980

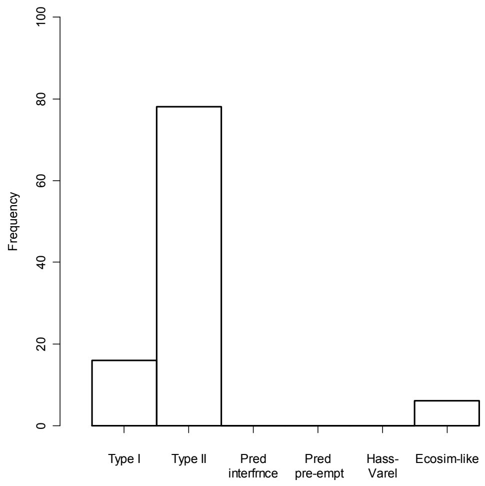


Figure 5.17. Frequency with which each of six alternative feeding functional relationships are selected using AIC for operating models based on a Type I feeding functional relationship when (a) the diet composition sample sizes are multiplied by four, and (b) the diet sample sizes are 100 for years and predator lengths since 1980.

6. EXTENSION AND FUTURE WORK

The major way in which this project could be extended would be to add additional predator and prey species (although this might increase the computational demands of the calculations to the point that they are essentially infeasible given current computing resources) and to examine sensitivity to various fixed inputs (e.g. *M* for cod) as well as how the data are weighted. The methods developed in the project can be extended in several other ways.

(1) Allowance could be made for each predator species to have a different feeding functional relationship. This extension will be necessary if additional predator species are included in the model, but the amount of diet data is even less for these species than for pollock, mackerel and cod in the Aleutians.

(2) The fits to the diet composition by length (Fig 4.10c) suggest that the ability to mimic the consumption of pollock by cod is poorer than desirable. It might be possible to rectify this concern by allowing each predator-prey interaction to be based on a different length-selectivity pattern (i.e. values for the parameters of Eqn 4.5).

(3) The model is fitted to daily ration in weight, but a more ideal way to represent consumption would be in terms of calorific needs – implementing this would, however, require a model of calorific content to be developed.

(4) The model is based on an annual time-step and ignores spatial considerations. In principle, the model time-step could be shortened (particularly if diet data were available seasonally) and allowance could be made for multiple spatial strata. Ultimately, whether such extensions are justified depends on the availability of suitable data.

(5) The model could be reparameterized to attempt improve the performance of the MCMC algorithm.

(6) The modelling framework could be used to provide the technical basis for evaluating the robustness of current harvest strategies to uncertainty related to multi-species interactions if management advice remains being based on single-species assessments (e.g. Goodman *et al*., 2002; Marasco *et al.* 2007).

(7) The analyses could be used to examine whether there are between-species correlations in various biologically parameters, such as recruitment, and, if so, priors could be placed on such correlations.

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8. ACKNOWLEDGEMENTS

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Appendix A : The Data Used in the Application to Pollock, Akta Mackerel and Pacific Cod in the Aleutian Islands

The data used in MLMAK were obtained from:

(1) The single-species assessments conducted using AMAK. AMAK data for walleye pollock (MODELA10_A.DAT, MODELA10_A.CTR) and Atka mackerel (ATKA03.DAT, ATKA03.CTR) in the Aleutians were provided respectively by Steve Barbeaux and Sandra Lowe (AFSC). The assessment authors also provided information on mean length-at-age and variance of length-at-age.

(2) Length-composition and other data for Pacific cod on the Aleutian Shelf from the AFSC Observer and Survey programs provided by Grant Thompson (AFSC).

(3) Diet data extracted from the AFSC REEM database on stomach samples for the three species in the Aleutians provided by Kerim Aydin (AFSC) and Geoff Lang (AFSC).

A.1 Data subject to measurement error

A.1.1 Catch biomass

Catch biomass (1000mt) by species from 1979 to 2003 are listed in Table A.1. Sources: pollock (column NRA w/o E of Table 1A.4 of Anon. (2003)); mackerel (column "total BSAI" of Table 15.1 of Anon. (2003) – includes discards and CDQ catches); and cod (Table 2.1b of Anon. (2003)). Catch data are provided for six cod fisheries in Anon (2003). These catches are combined into catches by three fisheries when applying MLMAK: a) trawls (foreign + joint venture + domestic), b) longline (foreign + domestic), and c) pots (domestic). Catch data are also available for "other" fisheries for cod, but the magnitude of these catches is small (1-3% of the total in 5 of the 25 years; zero otherwise) and so these catches were ignored for the analyses of this report.

A.1.2 Fishery age- and length-compositions

Fishery age-compositions (expressed as proportions) and the associated effective sample sizes[1] for pollock and mackerel were obtained from the assessment authors (Tables A.2 and A.3). The fishery length-compositions for cod (expressed as proportions) (Tables A.4-A.6) were calculated from data collected by the AFSC observer program and supplied by Grant Thomson (AFSC).

A.1.3 Survey indices of abundance

The survey indices of abundance and their standard errors are listed in Table A.7. Sources: pollock (Table 1A.9 of Anon. (2003)); mackerel (Tables 15.4 and 15.5 of Anon. (2003)); cod (Grant Thompson, AFSC, pers. commn).

A.1.4 Survey age- and length-compositions

Survey age-compositions (expressed as proportions) and the associated effective sample sizes for pollock and mackerel were obtained from the assessment authors (Tables A.8

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[1] The effective sample sizes for the walleye pollock and Atka mackerel fisheries and surveys were taken from the single-species assessments, while the effective sample sizes for Pacific cod were assigned.

and A.9). The survey length-compositions for cod (expressed as proportions) (Table A.10) were calculated from data supplied by Grant Thomson (AFSC).

A.1.5 Daily ration

Table A.11 lists the daily weight of prey required to support a predator of a given age. The values in Table A.11 were calculated from von Bertalanffy parameters (Essington *et al*., 2001) that were estimated by fitting a growth curve to data on population weight-at-age. The daily consumption rate of an individual predator of age t in kg prey/kg predator, C_t, can be calculated using the Equation:

$$C_t = \frac{k\left(W_\infty\right)^{1-d} W_{t+0.5} A^{-1}}{365} \tag{A.1}$$

where k and W_∞ are estimated by fitting the von Bertalanffy growth function in weight to observed population weights-at-age, i.e.

$$\hat{W}_t = W_\infty\left(1-e^{-k(1-d)(t-t_0)}\right)^{1/(1-d)} \tag{A.2}$$

$W_{t+0.5}$ is the weight of animal of age t in the middle of the year,
t_0 is the theoretical age at which weight is zero,
d is a scaling factor representing how metabolic rate changes with size, and
A is the proportion of food ingested that is assimilated.

A.1.6 Proportions of prey by weight

The numbers of fish sampled for stomach contents by year and predator species, and the split of the length-specific diet by prey-species, are listed in Tables A.12-A.17. These data were extracted from the AFSC REEM Food Habits database.

A.1.7 Proportions of prey by length

The numbers of predator stomachs sampled for lengths of prey by predator and prey species length-class are listed in Tables A.18-A.23. These data were extracted from the AFSC REEM Food Habits database.

A.2 Data assumed known without error

A.2.1 Fishery weights-at-age

Tables A.24-A.26 list the fishery weights-at-age. The fishery weights-at-age are year-specific for pollock, but time-invariant for mackerel and cod. The data for pollock and mackerel were taken from Tables 1A.6 and 15.6 of Anon (2003). Mean weights-at-age for the cod fisheries were calculated from the relationships between length and age, and weight and length (Anon (2003), pg. 132).

A.2.2 Surveys weights-at-age

Tables A.27 and A.28 list the survey weights-at-age for pollock and mackerel. Sources: pollock (Steve Barbeaux, AFSC, pers. commn) and mackerel (Table 15.6 of Anon.

(2003)). The survey weights-at-age for cod are assumed to be same as the fishery weights-at-age (see Table A.26).

A.2.3 Population weight- and maturity-at-age

Table A.29 lists the population weight-at-age and maturity-at-age vectors. Sources: (pollock maturity Table 1A.13 of Anon. (2003); pollock population weight-at-age Steve Barbeaux, AFSC, pers. commn; mackerel maturity Table 15.7 of Anon. (2003) and Sandra Lowe, AFSC, pers commn; mackerel population weights-at-age are set to the fishery weights-at-age in Table 15.6 of Anon (2003); cod population weights-at-age are set to the weights-at-age in the surveys and fisheries.

A.2.4 Age-length transition matrices

Tables A.30-A.32 list the length-at-age distributions (by 5cm length bins) for the three species. These matrices were calculated for each species from the mean length at each age and the associated standard deviations, assuming that length-at-age is normally distributed. The mean lengths-at-age for pollock were calculated for each age as the average of the mean lengths-at-age from surveys conducted during 1997, 2000, 2002 and 2004 (Steve Barbeaux, AFSC, pers. commn). The standard deviation of length-at-age for each age was set to that for the year for which the mean length-at-age for that age was the highest. The mean lengths-at-age for mackerel and their associated standard deviations were calculated using the raw length-age data obtained from surveys conducted during 1991, 1994, 1997, 2000, 2002 and 2004 (Sandra Lowe, AFSC, pers. commn). Mean length-at-age and its variance for cod were obtained from pg. 132 of Anon. (2003).

Table A.1. Catch biomass (1000t)

	Year																								
Fishery	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
pollock	6.4	31.0	23.0	20.0	17.2	6.3	0.9	0.7	2.7	0.6	0.0	10.5	0.6	8.5	16.2	6.0	58.0	23.0	25.8	23.3	0.6	0.9	0.6	0.3	0.3
mackerel	23.3	20.5	19.7	19.9	11.7	36.1	37.9	32.0	30.1	22.1	18.0	22.2	26.7	50.0	65.7	69.6	81.6	103.9	65.8	58.3	56.2	47.2	61.6	45.6	60.0
cod: trawl	0.0	0.0	7.2	7.9	8.0	7.2	6.1	6.9	13.1	5.0	4.2	6.9	3.4	14.6	17.3	14.4	10.6	21.2	17.3	20.5	16.4	20.4	15.8	27.9	27.7
cod: pot	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	3.2	6.3	0.0	0.1	1.0	4.6	0.6	0.4	3.8	3.1	0.5	0.0	0.0
cod: longline	0.0	0.0	0.3	0.5	0.4	0.8	0.8	0.0	0.0	0.1	0.3	0.6	3.2	22.1	16.9	7.0	4.9	5.8	7.2	13.8	7.9	16.2	17.8	2.9	0.9

Table A.2. The fishery proportions-at-age for pollock.

	Effective	Age (years)															
Year	sample size	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1990	2	0.000	0.000	0.000	0.088	0.021	0.143	0.092	0.124	0.126	0.033	0.178	0.031	0.082	0.033	0.017	0.034
1991	2	0.000	0.000	0.000	0.000	0.017	0.014	0.100	0.029	0.037	0.116	0.012	0.033	0.097	0.245	0.064	0.237
1992	2	0.000	0.000	0.000	0.000	0.003	0.039	0.072	0.034	0.077	0.016	0.088	0.061	0.065	0.048	0.233	0.265
1993	2	0.000	0.000	0.000	0.000	0.014	0.034	0.087	0.104	0.041	0.112	0.062	0.065	0.062	0.030	0.055	0.335
1994	2	0.000	0.000	0.000	0.000	0.013	0.200	0.061	0.082	0.111	0.086	0.062	0.027	0.079	0.021	0.021	0.240
1995	12	0.000	0.000	0.000	0.005	0.009	0.000	0.249	0.029	0.126	0.119	0.035	0.058	0.052	0.094	0.019	0.205
1996	12	0.000	0.000	0.000	0.008	0.008	0.032	0.086	0.308	0.092	0.124	0.072	0.056	0.038	0.012	0.046	0.119
1998	12	0.000	0.000	0.000	0.002	0.004	0.284	0.084	0.055	0.048	0.087	0.061	0.052	0.122	0.066	0.059	0.075

Table A.3. The fishery proportions-at-age for mackerel.

	Effective	Age (years)															
Year	sample size	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1979	25	0.000	0.000	0.000	0.094	0.563	0.273	0.046	0.023	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1980	25	0.000	0.000	0.000	0.446	0.208	0.254	0.059	0.021	0.008	0.005	0.000	0.000	0.000	0.000	0.000	0.000
1981	100	0.000	0.000	0.000	0.167	0.531	0.000	0.050	0.252	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1982	100	0.000	0.000	0.000	0.006	0.079	0.778	0.116	0.020	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1983	100	0.000	0.000	0.000	0.105	0.079	0.140	0.587	0.088	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1984	100	0.000	0.000	0.002	0.019	0.143	0.138	0.211	0.426	0.043	0.019	0.000	0.000	0.000	0.000	0.000	0.000
1985	100	0.000	0.000	0.011	0.268	0.148	0.158	0.101	0.092	0.196	0.021	0.005	0.000	0.000	0.000	0.000	0.000
1986	100	0.000	0.000	0.007	0.228	0.129	0.088	0.106	0.090	0.116	0.197	0.021	0.017	0.000	0.000	0.000	0.000
1987	100	0.000	0.000	0.014	0.269	0.196	0.118	0.049	0.061	0.056	0.044	0.174	0.014	0.006	0.000	0.000	0.000
1988	100	0.000	0.000	0.009	0.223	0.504	0.138	0.040	0.034	0.014	0.022	0.005	0.010	0.001	0.000	0.000	0.000
1990	100	0.000	0.000	0.000	0.150	0.448	0.252	0.093	0.033	0.007	0.005	0.002	0.001	0.001	0.006	0.001	0.000
1991	100	0.000	0.000	0.000	0.067	0.192	0.348	0.203	0.105	0.044	0.009	0.014	0.014	0.003	0.000	0.000	0.000
1994	100	0.000	0.000	0.000	0.090	0.066	0.226	0.375	0.043	0.089	0.062	0.040	0.006	0.003	0.000	0.000	0.000
1995	100	0.000	0.000	0.002	0.149	0.323	0.077	0.116	0.216	0.028	0.031	0.042	0.016	0.000	0.000	0.000	0.000
1996	100	0.000	0.000	0.000	0.021	0.394	0.134	0.077	0.125	0.191	0.018	0.022	0.016	0.003	0.000	0.000	0.000
1998	100	0.000	0.000	0.000	0.106	0.177	0.162	0.299	0.109	0.039	0.036	0.052	0.004	0.008	0.007	0.000	0.000
1999	100	0.000	0.000	0.016	0.013	0.496	0.124	0.099	0.143	0.057	0.020	0.014	0.014	0.002	0.002	0.000	0.000
2000	100	0.000	0.000	0.009	0.131	0.086	0.401	0.117	0.064	0.125	0.032	0.014	0.009	0.005	0.005	0.000	0.000
2001	100	0.000	0.000	0.019	0.254	0.138	0.090	0.297	0.088	0.050	0.048	0.009	0.004	0.001	0.001	0.000	0.000
2002	100	0.000	0.000	0.026	0.284	0.295	0.083	0.042	0.180	0.035	0.018	0.029	0.004	0.003	0.000	0.000	0.000

Table A.4. Trawl fishery proportions-at-length for cod.

		Length bin (5 cm midpoint)																						
Year	Effective sample size	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5	82.5	87.5	92.5	97.5	102.5	107.5	112.5	117.5
1990	100	0.000	0.000	0.000	0.000	0.000	0.002	0.010	0.013	0.053	0.104	0.094	0.123	0.159	0.117	0.113	0.100	0.056	0.029	0.022	0.004	0.001	0.000	0.000	0.000
1991	100	0.000	0.000	0.000	0.000	0.002	0.012	0.014	0.026	0.055	0.059	0.094	0.124	0.132	0.137	0.109	0.104	0.072	0.041	0.012	0.002	0.003	0.001	0.000	0.000
1992	100	0.000	0.000	0.000	0.000	0.000	0.003	0.011	0.032	0.050	0.043	0.059	0.090	0.117	0.152	0.151	0.117	0.085	0.050	0.029	0.009	0.003	0.001	0.000	0.000
1993	100	0.000	0.000	0.000	0.000	0.001	0.006	0.009	0.013	0.022	0.038	0.047	0.049	0.062	0.077	0.105	0.137	0.143	0.119	0.089	0.055	0.023	0.006	0.001	0.000
1994	100	0.000	0.000	0.001	0.000	0.000	0.000	0.001	0.005	0.008	0.022	0.045	0.068	0.076	0.115	0.110	0.126	0.133	0.117	0.099	0.049	0.019	0.004	0.001	0.000
1995	100	0.000	0.000	0.000	0.007	0.032	0.036	0.011	0.011	0.012	0.015	0.025	0.035	0.055	0.088	0.119	0.110	0.107	0.101	0.096	0.070	0.050	0.018	0.002	0.000
1996	100	0.000	0.000	0.000	0.001	0.004	0.005	0.013	0.026	0.025	0.028	0.030	0.037	0.055	0.076	0.107	0.131	0.140	0.121	0.096	0.060	0.034	0.012	0.001	0.001
1997	100	0.000	0.000	0.000	0.000	0.001	0.004	0.004	0.003	0.009	0.019	0.036	0.045	0.052	0.066	0.095	0.115	0.128	0.134	0.126	0.091	0.049	0.018	0.004	0.001
1998	100	0.000	0.000	0.000	0.000	0.001	0.008	0.013	0.011	0.014	0.020	0.030	0.056	0.087	0.116	0.112	0.105	0.096	0.099	0.098	0.078	0.042	0.013	0.002	0.000
1999	100	0.000	0.000	0.000	0.000	0.001	0.005	0.007	0.018	0.032	0.038	0.042	0.049	0.066	0.103	0.136	0.149	0.127	0.078	0.068	0.052	0.025	0.006	0.001	0.000
2000	100	0.000	0.000	0.000	0.000	0.001	0.002	0.003	0.013	0.021	0.030	0.046	0.062	0.071	0.095	0.106	0.119	0.139	0.123	0.079	0.056	0.027	0.006	0.001	0.000
2001	100	0.000	0.000	0.000	0.000	0.002	0.004	0.003	0.003	0.005	0.016	0.036	0.075	0.122	0.170	0.134	0.112	0.101	0.080	0.074	0.042	0.018	0.004	0.001	0.000
2002	100	0.000	0.000	0.000	0.000	0.001	0.003	0.007	0.009	0.010	0.014	0.017	0.029	0.074	0.143	0.173	0.170	0.134	0.089	0.061	0.044	0.020	0.003	0.000	0.000
2003	100	0.000	0.000	0.000	0.000	0.001	0.004	0.005	0.014	0.023	0.027	0.036	0.044	0.066	0.083	0.105	0.150	0.161	0.128	0.085	0.045	0.019	0.004	0.001	0.000

Table A.5. Pot fishery proportions-at-length for cod.

		Length bin (5 cm midpoint)																						
Year	Effective sample size	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5	82.5	87.5	92.5	97.5	102.5	107.5	112.5	117.5
1991	200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.005	0.017	0.052	0.117	0.160	0.182	0.185	0.146	0.082	0.041	0.011	0.001	0.000	0.000	0.000
1992	200	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.012	0.036	0.053	0.071	0.099	0.134	0.147	0.149	0.126	0.089	0.048	0.022	0.009	0.004	0.001	0.000	0.000
1994	200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.044	0.056	0.111	0.144	0.122	0.122	0.144	0.144	0.067	0.011	0.022	0.011	0.000	0.000	0.000
1995	200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.014	0.036	0.066	0.093	0.113	0.121	0.118	0.123	0.110	0.109	0.051	0.030	0.009	0.002	0.000	0.000
1996	200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.026	0.034	0.056	0.090	0.111	0.116	0.117	0.119	0.105	0.086	0.069	0.045	0.018	0.003	0.001	0.000
1997	200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.008	0.008	0.045	0.080	0.091	0.102	0.098	0.106	0.106	0.099	0.098	0.073	0.053	0.024	0.005	0.000
1998	200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.021	0.049	0.106	0.122	0.125	0.116	0.093	0.092	0.081	0.087	0.063	0.032	0.010	0.000	0.000
1999	200	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.008	0.025	0.045	0.077	0.101	0.108	0.113	0.114	0.106	0.097	0.078	0.059	0.044	0.018	0.005	0.000	0.000
2000	200	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.008	0.030	0.045	0.075	0.072	0.078	0.091	0.098	0.112	0.128	0.090	0.090	0.040	0.030	0.012	0.000	0.001
2001	200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.011	0.037	0.063	0.113	0.173	0.157	0.116	0.094	0.072	0.055	0.050	0.025	0.021	0.011	0.000	0.000

Table A.6. Longline fishery proportions-at-length for cod.

	Effective	Length bin (5 cm midpoint)																						
Year	sample size	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5	82.5	87.5	92.5	97.5	102.5	107.5	112.5	117.5
1990	200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.020	0.055	0.123	0.164	0.153	0.169	0.144	0.099	0.043	0.019	0.009	0.001	0.000	0.000	0.000
1991	200	0.000	0.000	0.000	0.000	0.000	0.001	0.005	0.015	0.029	0.060	0.097	0.117	0.129	0.130	0.126	0.103	0.086	0.057	0.030	0.013	0.002	0.000	0.000	0.000
1992	200	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.021	0.040	0.051	0.064	0.083	0.109	0.126	0.133	0.120	0.097	0.071	0.048	0.022	0.009	0.002	0.000	0.000
1993	200	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.015	0.048	0.089	0.097	0.087	0.093	0.111	0.118	0.116	0.094	0.064	0.038	0.020	0.005	0.001	0.000	0.000
1994	200	0.000	0.000	0.000	0.000	0.000	0.001	0.005	0.019	0.044	0.065	0.095	0.117	0.126	0.115	0.098	0.084	0.085	0.065	0.046	0.024	0.007	0.002	0.001	0.000
1995	200	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.004	0.013	0.032	0.056	0.088	0.112	0.130	0.129	0.120	0.093	0.080	0.072	0.047	0.017	0.004	0.000	0.000
1996	200	0.000	0.000	0.000	0.000	0.000	0.003	0.009	0.034	0.065	0.078	0.083	0.105	0.119	0.115	0.105	0.094	0.073	0.041	0.033	0.024	0.017	0.005	0.000	0.000
1997	200	0.000	0.000	0.000	0.000	0.000	0.002	0.005	0.014	0.040	0.069	0.100	0.099	0.088	0.093	0.098	0.099	0.088	0.082	0.058	0.038	0.021	0.006	0.001	0.000
1998	200	0.000	0.000	0.000	0.000	0.001	0.002	0.008	0.027	0.052	0.062	0.073	0.096	0.121	0.121	0.103	0.081	0.072	0.061	0.055	0.037	0.020	0.006	0.001	0.000
1999	200	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.024	0.051	0.078	0.087	0.103	0.102	0.113	0.107	0.101	0.073	0.052	0.043	0.034	0.020	0.006	0.001	0.000
2000	200	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.019	0.040	0.063	0.092	0.104	0.114	0.106	0.091	0.085	0.083	0.071	0.053	0.041	0.026	0.007	0.001	0.000
2001	200	0.000	0.000	0.000	0.000	0.000	0.001	0.009	0.027	0.050	0.078	0.103	0.126	0.136	0.120	0.092	0.069	0.051	0.042	0.039	0.031	0.019	0.006	0.001	0.000
2002	200	0.000	0.000	0.000	0.000	0.000	0.003	0.015	0.028	0.061	0.074	0.071	0.105	0.143	0.159	0.138	0.083	0.046	0.029	0.019	0.013	0.008	0.004	0.001	0.000
2003	200	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.022	0.076	0.140	0.148	0.133	0.124	0.096	0.083	0.065	0.048	0.031	0.018	0.007	0.003	0.001	0.000	0.000

Table A.7. The survey estimates of abundance and their sampling standard errors.

	Walleye pollock		Atka mackerel		Pacific cod	
Survey year	Estimate	Standard error	Estimate	Standard error	Estimate	Standard error
1980					148.27	29.78
1983					215.76	30.99
1986			544.75	343.20	255.01	66.52
1991	83.3	16.8	723.92	104.89	189.19	25.53
1994	47.6	9.7	602.16	198.15	184.11	33.70
1997	57.6	15.7	366.31	105.47	83.42	10.50
2000	76.6	27.5	510.86	144.78	136.08	23.55
2002	121.9	55.1	772.80	157.26	82.85	12.02

Table A.8. Survey proportions-at-age for pollock.

Year	Effective sample size	Age (years)															
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1991	200	0.000	0.091	0.142	0.248	0.112	0.034	0.044	0.028	0.070	0.032	0.045	0.019	0.060	0.040	0.037	0.000
1997	200	0.019	0.018	0.029	0.107	0.129	0.101	0.084	0.158	0.098	0.040	0.047	0.030	0.027	0.016	0.096	0.000
2002	200	0.017	0.015	0.044	0.042	0.047	0.076	0.108	0.088	0.104	0.085	0.090	0.071	0.092	0.041	0.081	0.000

Table A.9. Survey proportions-at-age for mackerel.

Year	Effective sample size	Age (years)															
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1986	50	0.000	0.000	0.058	0.363	0.196	0.127	0.101	0.085	0.050	0.015	0.004	0.001	0.000	0.000	0.000	0.000
1991	50	0.000	0.000	0.048	0.561	0.091	0.173	0.054	0.058	0.010	0.004	0.000	0.000	0.000	0.000	0.000	0.000
1994	50	0.000	0.000	0.014	0.188	0.130	0.210	0.246	0.058	0.077	0.025	0.043	0.006	0.000	0.000	0.001	0.000
1997	50	0.000	0.000	0.102	0.222	0.179	0.231	0.071	0.036	0.049	0.067	0.010	0.010	0.003	0.008	0.000	0.000
2000	50	0.000	0.000	0.288	0.082	0.027	0.242	0.073	0.076	0.127	0.040	0.020	0.015	0.006	0.003	0.000	0.001

Table A.10. Survey proportions-at-length for cod.

Year	Effective sample size	Length (cm)																						
		2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5	82.5	87.5	92.5	97.5	102.5	107.5	112
1991	100	0.000	0.000	0.002	0.001	0.033	0.172	0.148	0.051	0.046	0.046	0.071	0.081	0.105	0.083	0.067	0.044	0.027	0.017	0.005	0.002	0.001	0.000	0.0
1994	100	0.000	0.095	0.231	0.014	0.003	0.022	0.028	0.037	0.062	0.045	0.051	0.074	0.077	0.075	0.049	0.041	0.040	0.034	0.017	0.005	0.001	0.001	0.0
1997	100	0.000	0.000	0.016	0.100	0.018	0.035	0.061	0.034	0.059	0.091	0.147	0.146	0.096	0.057	0.037	0.028	0.024	0.021	0.017	0.011	0.004	0.000	0.0
2000	100	0.000	0.008	0.027	0.002	0.007	0.015	0.018	0.062	0.109	0.130	0.153	0.156	0.113	0.077	0.054	0.026	0.018	0.011	0.007	0.003	0.004	0.001	0.0
2002	100	0.000	0.003	0.019	0.015	0.026	0.096	0.146	0.075	0.095	0.094	0.095	0.051	0.042	0.049	0.055	0.061	0.039	0.022	0.008	0.007	0.002	0.001	0.0

Table A.11. Daily ration.

Species	Age (years)														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Pollock	0.0152	0.0109	0.0095	0.0086	0.0082	0.0078	0.0076	0.0075	0.0075	0.0074	0.0073	0.0072	0.0073	0.0073	0.0073
Mackerel	0.0365	0.0348	0.0274	0.0250	0.0244	0.0241	0.0240	0.0240	0.0240	0.0240	0.0240	0.0240	0.0240	0.0240	0.0240
Cod	0.0061	0.0049	0.0042	0.0035	0.0032	0.0030	0.0028	0.0026	0.0026	0.0025	0.0025	0.0024			

Table A.12. Number of pollock stomachs sampled for prey weights by length bin and sampling year.

Pollock length (cm)	Year																
	1982	1983	1986	1987	1988	1990	1991	1992	1993	1994	1995	1996	1997	2000	2001	2002	2003
2.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0
12.5	0	0	0	0	0	0	0	0	0	13	0	0	20	106	23	69	0
17.5	0	0	0	0	0	0	128	0	0	234	0	0	108	70	19	51	3
22.5	0	0	0	4	0	0	0	0	0	0	0	0	50	0	47	2	15
27.5	0	0	0	1	0	0	68	0	0	62	3	0	34	44	21	46	9
32.5	0	0	1	4	9	0	235	0	0	30	0	0	56	28	11	40	18
37.5	9	105	30	17	11	61	287	2	0	97	14	0	23	60	4	56	16
42.5	8	0	153	188	14	85	400	0	0	166	8	0	46	112	13	109	8
47.5	0	28	211	342	95	78	706	2	14	431	1	1	152	226	8	205	6
52.5	10	46	86	342	71	99	574	14	31	397	8	2	233	334	4	250	8
57.5	0	32	14	104	14	45	245	17	21	305	4	0	221	392	6	295	7
62.5	0	29	2	10	3	4	56	2	3	92	4	0	70	176	0	170	1
67.5	0	11	0	1	0	0	20	0	2	24	0	0	3	28	0	32	0

Table A.13. Fraction of total prey weight of pollock that consists of each prey species.

(a) pollock as prey of pollock

Predator length (cm)	Year																
	1982	1983	1986	1987	1988	1990	1991	1992	1993	1994	1995	1996	1997	2000	2001	2002	2003
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
27.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.306	0.000	0.000
32.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
37.5	0.000	0.000	0.134	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.352
42.5	0.000	0.000	0.129	0.098	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.272	0.000	0.944
47.5	0.000	0.000	0.348	0.131	0.000	0.040	0.000	0.000	0.000	0.021	0.000	0.000	0.000	0.000	0.319	0.000	0.000
52.5	0.000	0.000	0.561	0.148	0.099	0.010	0.004	0.000	0.000	0.019	0.000	0.000	0.000	0.000	0.902	0.000	0.658
57.5	0.000	0.000	0.620	0.109	0.000	0.138	0.015	0.000	0.000	0.025	0.000	0.000	0.000	0.000	0.221	0.000	0.410
62.5	0.000	0.000	0.947	0.565	0.000	0.476	0.002	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000
67.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.317	0.000	0.000	0.000	0.000	0.000	0.000	0.000

(b) mackerel as prey of pollock

Predator length (cm)	Year																
	1982	1983	1986	1987	1988	1990	1991	1992	1993	1994	1995	1996	1997	2000	2001	2002	2003
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
27.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
32.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
37.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.105	0.000	0.000	0.000	0.000	0.000	0.000	0.000
42.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
47.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
52.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.016	0.000	0.000	0.000	0.000	0.000	0.030	0.000
57.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.023	0.000	0.000	0.000	0.000	0.000	0.026	0.000
62.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.000
67.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.465	0.000	0.000	0.000	0.000	0.000	0.179	0.000

(c) cod as prey of pollock

Predator length (cm)	Year																
	1982	1983	1986	1987	1988	1990	1991	1992	1993	1994	1995	1996	1997	2000	2001	2002	2003
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
27.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
32.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
37.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
42.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
47.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
52.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
57.5	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
62.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
67.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

(d) non-modeled species as prey of pollock

Predator length (cm)	Year																
	1982	1983	1986	1987	1988	1990	1991	1992	1993	1994	1995	1996	1997	2000	2001	2002	2003
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
12.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	1.000	1.000	1.000	1.000	0.000
17.5	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000
22.5	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1.000	1.000	1.000
27.5	0.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	1.000	0.000	1.000	1.000	0.694	1.000	1.000
32.5	0.000	0.000	1.000	1.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000
37.5	1.000	1.000	0.866	1.000	1.000	1.000	1.000	1.000	0.000	0.895	1.000	0.000	1.000	1.000	1.000	1.000	0.648
42.5	1.000	0.000	0.871	0.902	0.963	1.000	1.000	0.000	0.000	1.000	1.000	0.000	1.000	1.000	0.728	1.000	0.056
47.5	0.000	1.000	0.652	0.869	1.000	0.960	1.000	1.000	1.000	0.979	1.000	1.000	1.000	1.000	0.681	1.000	1.000
52.5	1.000	1.000	0.439	0.852	0.901	0.990	0.996	1.000	1.000	0.966	1.000	1.000	1.000	1.000	0.098	0.970	0.342
57.5	0.000	1.000	0.380	0.891	1.000	0.862	0.984	1.000	1.000	0.952	1.000	0.000	1.000	1.000	0.779	0.974	0.590
62.5	0.000	1.000	0.053	0.435	1.000	0.524	0.998	1.000	1.000	0.958	1.000	0.000	1.000	1.000	0.000	0.990	1.000
67.5	0.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	1.000	0.219	0.000	0.000	1.000	1.000	0.000	0.821	0.000

Table A.14. Number of mackerel stomachs sampled for prey weights by length bin and sampling year.

Predator length (cm)	Year										
	1986	1987	1988	1990	1991	1994	1997	1999	2000	2001	2002
2.5	0	0	0	0	0	0	0	0	0	0	0
7.5	0	0	0	0	0	0	0	0	0	0	0
12.5	0	0	0	1	0	0	0	0	15	0	0
17.5	0	0	0	0	0	5	0	0	83	0	9
22.5	0	1	0	0	56	17	29	0	94	4	48
27.5	2	76	5	0	323	43	38	0	59	11	261
32.5	10	127	55	0	555	245	164	0	254	203	416
37.5	21	519	22	21	540	571	445	893	701	130	548
42.5	45	472	13	42	32	245	136	2317	352	181	295
47.5	1	10	0	12	0	46	14	150	30	32	9

Table A.15. Fraction of total prey weight of mackerel that consists of each prey species.

(a) pollock as prey of mackerel

Predator length (cm)	Year										
	1986	1987	1988	1990	1991	1994	1997	1999	2000	2001	2002
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
27.5	0.000	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.000	0.000	0.000
32.5	0.074	0.019	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000
37.5	0.637	0.000	0.000	0.000	0.036	0.000	0.000	0.000	0.000	0.000	0.000
42.5	0.523	0.000	0.000	0.000	0.749	0.000	0.000	0.000	0.000	0.000	0.000
47.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

(b) mackerel as prey of mackerel

Predator length (cm)	Year										
	1986	1987	1988	1990	1991	1994	1997	1999	2000	2001	2002
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
27.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
32.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
37.5	0.000	0.000	0.000	0.000	0.000	0.024	0.000	0.000	0.000	0.000	0.002
42.5	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.094	0.064
47.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

(c) cod as prey of mackerel

Predator length (cm)	Year										
	1986	1987	1988	1990	1991	1994	1997	1999	2000	2001	2002
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
27.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
32.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
37.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
42.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
47.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

(d) non-modeled species as prey of mackerel

Predator length (cm)	Year										
	1986	1987	1988	1990	1991	1994	1997	1999	2000	2001	2002
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22.5	0.000	1.000	0.000	0.000	1.000	1.000	1.000	0.000	1.000	1.000	1.000
27.5	1.000	1.000	1.000	0.000	0.995	1.000	1.000	0.000	1.000	1.000	1.000
32.5	0.926	0.981	1.000	0.000	0.998	1.000	1.000	0.000	1.000	1.000	1.000
37.5	0.363	1.000	1.000	1.000	0.964	0.976	1.000	1.000	1.000	1.000	0.998
42.5	0.477	1.000	1.000	1.000	0.251	0.998	1.000	1.000	1.000	0.906	0.936
47.5	1.000	1.000	0.000	1.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000

Table A.16. Number of cod stomachs sampled for prey weights by length bin and sampling year.

Predator length (cm)	Year																			
	1983	1984	1985	1986	1987	1988	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
2.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12.5	0	0	0	0	0	0	0	4	0	0	13	0	0	0	0	0	1	0	0	0
17.5	0	0	0	0	0	0	0	49	0	0	82	0	0	8	0	0	9	0	0	0
22.5	0	0	2	0	0	0	0	43	0	0	19	0	0	33	0	0	17	0	5	0
27.5	0	0	0	0	0	0	0	11	0	0	7	0	0	0	0	13	33	0	3	0
32.5	0	0	18	0	0	0	0	185	0	0	57	4	0	24	0	4	71	0	57	0
37.5	0	0	30	1	0	0	5	290	0	3	170	0	2	97	0	3	49	0	108	0
42.5	0	0	30	41	40	23	1	338	6	19	162	3	0	51	0	22	103	0	81	0
47.5	0	27	60	98	83	30	16	375	13	68	278	4	20	89	0	14	200	2	233	7
52.5	0	21	29	45	88	21	20	251	6	25	256	1	9	208	3	9	156	0	185	6
57.5	0	3	48	27	80	0	35	373	8	45	154	2	5	282	2	6	287	6	204	7
62.5	0	5	52	26	61	0	24	480	3	17	117	7	7	354	2	4	423	9	129	11
67.5	0	3	35	37	46	0	32	288	9	6	134	2	13	236	9	4	306	39	123	30
72.5	3	1	28	42	38	0	3	182	10	11	85	2	12	84	6	0	214	23	88	23
77.5	5	2	14	42	40	0	11	140	4	15	21	2	8	27	3	1	130	37	159	23
82.5	0	4	13	17	33	8	7	61	6	25	36	3	9	28	13	3	73	21	113	11
87.5	4	3	7	19	20	0	9	39	0	3	26	1	1	10	10	4	56	8	58	10
92.5	0	3	6	14	17	0	5	12	1	0	7	4	3	7	11	4	47	9	42	6
97.5	0	1	0	4	5	4	4	10	1	8	3	1	1	15	6	2	6	4	17	1
102.5	0	0	0	0	5	0	0	1	1	3	4	0	0	0	9	0	9	2	7	0
107.5	0	2	1	0	0	0	0	4	0	0	3	0	3	2	7	0	0	0	0	0
112.5	0	0	0	0	0	2	0	0	0	0	0	0	0	0	6	0	6	0	2	0
117.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table A.17. Fraction of total prey weight of cod that consists of each prey species

(a) pollock as prey of cod

Predator length (cm)	Year																			
	1983	1984	1985	1986	1987	1988	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
27.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
32.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
37.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
42.5	0.000	0.000	0.000	0.000	0.327	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
47.5	0.000	0.000	0.021	0.000	0.000	0.000	0.000	0.000	0.000	0.014	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
52.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.215	0.000	0.007	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
57.5	0.000	0.000	0.000	0.039	0.025	0.000	0.000	0.000	0.839	0.014	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
62.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.018	0.020	0.103	0.113	0.000	0.000	0.012	0.000	0.000	0.025	0.000	0.014	0.000
67.5	0.000	0.000	0.052	0.000	0.000	0.000	0.000	0.142	0.069	0.000	0.573	0.000	0.000	0.085	0.000	0.000	0.132	0.000	0.000	0.279
72.5	0.000	0.000	0.245	0.317	0.101	0.000	0.000	0.158	0.439	0.465	0.307	0.000	0.000	0.082	0.000	0.000	0.003	0.000	0.054	0.025
77.5	0.000	0.000	0.000	0.523	0.272	0.000	0.000	0.188	0.000	0.176	0.464	0.000	0.000	0.313	0.000	0.000	0.000	0.000	0.039	0.108
82.5	0.000	0.235	0.096	0.089	0.063	0.856	0.000	0.080	0.118	0.078	0.134	0.000	0.000	0.407	0.057	0.000	0.000	0.000	0.036	0.560
87.5	0.000	0.000	0.043	0.185	0.386	0.000	0.000	0.584	0.000	0.000	0.351	0.000	0.000	0.022	0.000	0.000	0.009	0.000	0.249	0.385
92.5	0.000	0.000	0.000	0.569	0.238	0.000	0.108	0.335	0.000	0.000	0.772	0.000	0.524	0.131	0.000	0.000	0.000	0.000	0.000	0.719
97.5	0.000	0.000	0.000	0.000	0.466	0.000	0.526	0.927	0.000	0.020	0.000	0.000	0.000	0.547	0.695	0.000	0.000	0.000	0.000	0.000
102.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000
107.5	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.180	0.000	0.000	0.000	0.000	0.783	0.000	0.000	0.000	0.000	0.000	0.000	0.000
112.5	0.000	0.000	0.000	0.000	0.000	0.962	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.509	0.000
117.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

(b) mackerel as prey of cod

Predator length (cm)	Year																			
	1983	1984	1985	1986	1987	1988	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
27.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
32.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
37.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
42.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.267	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
47.5	0.000	0.000	0.307	0.000	0.000	0.000	0.000	0.129	0.118	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.327	0.000	0.029	0.000
52.5	0.000	0.000	0.497	0.000	0.000	0.000	0.296	0.147	0.000	0.000	0.062	0.000	0.000	0.066	0.000	0.000	0.354	0.000	0.120	0.000
57.5	0.000	0.000	0.343	0.331	0.203	0.000	0.140	0.045	0.000	0.000	0.443	0.000	0.000	0.075	0.000	0.000	0.459	0.000	0.166	0.000
62.5	0.000	0.000	0.730	0.000	0.061	0.000	0.000	0.237	0.980	0.000	0.059	0.000	0.000	0.153	0.000	0.000	0.341	0.000	0.000	0.000
67.5	0.000	0.000	0.656	0.170	0.062	0.000	0.084	0.387	0.299	0.752	0.103	0.655	0.000	0.140	0.000	0.897	0.233	0.235	0.341	0.000
72.5	0.000	0.000	0.352	0.000	0.340	0.000	0.000	0.259	0.529	0.000	0.185	0.999	0.487	0.265	0.920	0.000	0.346	0.499	0.757	0.000
77.5	0.000	0.000	0.830	0.153	0.262	0.000	0.000	0.285	0.888	0.646	0.251	0.000	0.488	0.000	0.221	0.000	0.489	0.770	0.559	0.253
82.5	0.000	0.000	0.638	0.733	0.249	0.000	0.088	0.277	0.882	0.144	0.579	0.790	0.000	0.218	0.703	0.000	0.378	0.372	0.774	0.000
87.5	0.000	0.000	0.894	0.380	0.000	0.000	0.096	0.158	0.000	0.000	0.462	1.000	0.000	0.000	0.317	0.541	0.014	0.975	0.656	0.466
92.5	0.000	0.000	0.818	0.298	0.085	0.000	0.801	0.567	1.000	0.000	0.214	0.999	0.000	0.151	0.640	0.000	0.009	0.791	0.896	0.000
97.5	0.000	0.000	0.000	0.610	0.423	0.000	0.000	0.057	1.000	0.709	1.000	0.000	0.000	0.000	0.277	0.851	0.052	0.994	0.821	0.000
102.5	0.000	0.000	0.000	0.000	0.391	0.000	0.000	1.000	1.000	0.900	0.899	0.000	0.000	0.000	0.105	0.000	0.000	1.000	0.911	0.000
107.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.313	0.000	0.000	0.000	0.000	0.000
112.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.778	0.000	0.542	0.000	0.000	0.000
117.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

(c) cod as prey of cod

Predator length (cm)	Year																			
	1983	1984	1985	1986	1987	1988	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
27.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
32.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
37.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
42.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
47.5	0.000	0.000	0.000	0.000	0.014	0.000	0.000	0.000	0.611	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
52.5	0.000	0.000	0.000	0.000	0.019	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
57.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
62.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.017	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
67.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.166
72.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
77.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.019	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
82.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
87.5	0.000	0.000	0.062	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
92.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
97.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
102.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
107.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
112.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
117.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

(d) non-modeled species as prey of cod

Predator length (cm)	Year																			
	1983	1984	1985	1986	1987	1988	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
17.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	0.000
22.5	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	1.000	0.000
27.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1.000	1.000	0.000	1.000	0.000
32.5	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	1.000	1.000	0.000	1.000	0.000	1.000	1.000	0.000	1.000	0.000
37.5	0.000	0.000	1.000	1.000	0.000	0.000	1.000	1.000	0.000	1.000	1.000	0.000	1.000	1.000	0.000	1.000	1.000	0.000	1.000	0.000
42.5	0.000	0.000	1.000	1.000	0.673	1.000	1.000	1.000	1.000	1.000	0.731	1.000	0.000	0.999	0.000	1.000	1.000	0.000	1.000	0.000
47.5	0.000	1.000	0.672	1.000	0.986	1.000	1.000	0.871	0.271	0.986	1.000	1.000	1.000	1.000	0.000	1.000	0.673	1.000	0.971	1.000
52.5	0.000	1.000	0.503	1.000	0.981	1.000	0.704	0.853	0.785	1.000	0.932	1.000	1.000	0.934	1.000	1.000	0.646	0.000	0.880	1.000
57.5	0.000	1.000	0.657	0.629	0.773	0.000	0.860	0.955	0.161	0.986	0.557	1.000	1.000	0.925	1.000	1.000	0.541	1.000	0.834	1.000
62.5	0.000	1.000	0.270	1.000	0.939	0.000	1.000	0.727	0.000	0.897	0.828	1.000	1.000	0.835	1.000	1.000	0.634	1.000	0.986	1.000
67.5	0.000	1.000	0.292	0.830	0.938	0.000	0.916	0.471	0.632	0.248	0.324	0.345	1.000	0.775	1.000	0.103	0.635	0.765	0.659	0.555
72.5	1.000	1.000	0.403	0.683	0.559	0.000	1.000	0.583	0.031	0.535	0.507	0.001	0.513	0.654	0.080	0.000	0.652	0.501	0.189	0.975
77.5	1.000	1.000	0.170	0.324	0.466	0.000	1.000	0.508	0.112	0.178	0.285	1.000	0.512	0.687	0.779	1.000	0.510	0.230	0.401	0.639
82.5	0.000	0.765	0.266	0.178	0.688	0.144	0.912	0.644	0.000	0.778	0.287	0.210	1.000	0.375	0.240	1.000	0.622	0.628	0.190	0.440
87.5	1.000	1.000	0.001	0.435	0.614	0.000	0.904	0.258	0.000	1.000	0.187	0.000	1.000	0.978	0.683	0.459	0.977	0.025	0.095	0.150
92.5	0.000	1.000	0.182	0.133	0.678	0.000	0.090	0.098	0.000	0.000	0.014	0.001	0.476	0.718	0.360	1.000	0.991	0.209	0.104	0.281
97.5	0.000	1.000	0.000	0.390	0.111	1.000	0.474	0.016	0.000	0.271	0.000	1.000	1.000	0.453	0.028	0.149	0.948	0.006	0.179	1.000
102.5	0.000	0.000	0.000	0.000	0.609	0.000	0.000	0.000	0.000	0.100	0.101	0.000	0.000	0.000	0.858	0.000	1.000	0.000	0.089	0.000
107.5	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.820	0.000	0.000	0.000	0.000	0.217	1.000	0.687	0.000	0.000	0.000	0.000	0.000
112.5	0.000	0.000	0.000	0.000	0.000	0.038	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.222	0.000	0.458	0.000	0.491	0.000
117.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table A.18. Number of pollock stomachs sampled for prey lengths that contained each prey species (by predator length-class and year)

Predator length (cm)	Pollock								Mackerel			Cod
	1986	1987	1988	1990	1991	1994	2001	2003	1994	2000	2002	1991
2.5	0	0	0	0	0	0	0	0	0	0	0	0
7.5	0	0	0	0	0	0	0	0	0	0	0	0
12.5	0	0	0	0	0	0	0	0	0	0	0	0
17.5	0	0	0	0	0	0	0	0	0	0	0	0
22.5	0	0	0	0	0	0	0	0	0	0	0	0
27.5	0	0	0	0	0	0	3	0	0	0	0	0
32.5	0	0	0	0	0	0	0	0	0	0	0	0
37.5	4	0	0	0	0	0	0	2	0	0	0	0
42.5	13	5	1	0	0	0	3	6	0	0	0	0
47.5	61	12	0	0	0	4	2	0	0	0	0	0
52.5	43	3	3	1	1	2	3	1	1	0	1	0
57.5	4	3	0	1	3	4	3	1	0	0	0	1
62.5	2	3	0	1	0	6	0	0	0	0	1	0
67.5	0	0	0	0	0	2	0	0	1	0	1	0

Table A.19. Breakdown of the diet of each prey species by pollock of various lengths (N/A denotes cases in which the prey length exceeds the predator length)

(a) Pollock predation on pollock

Predator length	Prey length												
	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5
27.5	0.000	0.667	0.333	0.000	0.000	0.000	N/A	N/A	N/A	N/A	N/A	N/A	N/A
37.5	0.667	0.000	0.333	0.000	0.000	0.000	0.000	N/A	N/A	N/A	N/A	N/A	N/A
42.5	0.214	0.357	0.393	0.036	0.000	0.000	0.000	0.000	N/A	N/A	N/A	N/A	N/A
47.5	0.342	0.595	0.063	0.000	0.000	0.000	0.000	0.000	0.000	N/A	N/A	N/A	N/A
52.5	0.333	0.579	0.018	0.035	0.035	0.000	0.000	0.000	0.000	0.000	N/A	N/A	N/A
57.5	0.421	0.368	0.158	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.000	N/A	N/A
62.5	0.500	0.250	0.250	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	N/A

(b) Pollock predation on mackerel

Predator length	Prey length									
	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5
52.5	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
62.5	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
62.7	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000

(c) Pollock predation on cod

Predator	Prey length										
length	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5+
57.5	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	N/A

Table A.20. Number of mackerel stomachs sampled for prey lengths that contained each prey species (by predator length-class and year)

Predator length (cm)	Pollock			Mackerel	Cod
	1986	1987	1991	2001	1994
2.5	0	0	0	0	0
7.5	0	0	0	0	0
12.5	0	0	0	0	0
17.5	0	0	0	0	0
22.5	0	0	0	0	0
27.5	0	0	1	0	0
32.5	3	2	1	0	0
37.5	31	0	7	0	1
42.5	42	0	2	1	0
47.5	0	0	0	0	0

Table A.21. Breakdown of the diet of each prey species by mackerel of various lengths (N/A denotes cases in which the prey length exceeds the predator length)

(a) mackerel predation on pollock

Predator	Prey length								
length	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5+
27.5	0.000	1.000	0.000	0.000	0.000	N/A	N/A	N/A	N/A
32.5	0.833	0.000	0.000	0.167	0.000	0.000	N/A	N/A	N/A
37.5	0.737	0.263	0.000	0.000	0.000	0.000	0.000	N/A	N/A
42.5	0.386	0.591	0.023	0.000	0.000	0.000	0.000	0.000	N/A

(b) mackerel predation on mackerel

Predator	Prey length								
length	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5+
42.5	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	N/A

(c) mackerel predation on cod

Predator	Prey length							
length	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5+
37.5	1.000	0.000	0.000	0.000	0.000	0.000	0.000	N/A

Table A.22. Number of cod stomachs sampled for prey lengths that contained each prey species (by predator length-class and year)
(a) cod predation on pollock

Predator length (cm)	Year 1985	1986	1987	1988	1990	1991	1993	1994	1997	1998	2000	2002	2003
2.5	0	0	0	0	0	0	0	0	0	0	0	0	0
7.5	0	0	0	0	0	0	0	0	0	0	0	0	0
12.5	0	0	0	0	0	0	0	0	0	0	0	0	0
17.5	0	0	0	0	0	0	0	0	0	0	0	0	0
22.5	0	0	0	0	0	0	0	0	0	0	0	0	0
27.5	0	0	0	0	0	0	0	0	0	0	0	0	0
32.5	0	0	0	0	0	0	0	0	0	0	0	0	0
37.5	0	0	0	0	0	0	0	0	0	0	0	0	0
42.5	0	0	1	0	0	0	0	0	0	0	0	0	0
47.5	0	0	0	0	0	0	1	0	0	0	0	0	0
52.5	0	0	0	0	0	0	0	0	0	0	0	0	0
57.5	0	0	0	0	0	0	1	0	0	0	0	0	0
62.5	0	0	0	0	0	4	0	0	4	0	1	1	0
67.5	0	0	0	0	0	4	0	9	1	0	1	0	1
72.5	0	2	0	0	0	4	3	2	1	0	0	0	1
77.5	0	2	2	0	0	5	0	3	1	0	0	0	1
82.5	0	0	0	3	0	1	1	2	1	0	0	0	2
87.5	0	0	0	0	0	3	0	4	0	0	0	6	1
92.5	0	1	2	0	1	2	0	2	0	0	0	0	2
97.5	0	0	1	0	1	2	1	0	1	1	0	0	0
102.5	0	0	0	0	0	0	0	0	0	0	0	0	0
107.5	1	0	0	0	0	0	0	0	0	0	0	0	0
112.5	0	0	0	1	0	0	0	0	0	0	0	0	0
117.5	0	0	0	0	0	0	0	0	0	0	0	0	0

(b) cod predation on mackerel

Predator length (cm)	Year														
	1985	1986	1987	1990	1991	1993	1994	1995	1996	1997	1998	2000	2001	2002	2003
2.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42.5	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0
47.5	1	0	0	0	1	0	0	0	0	0	0	1	0	1	0
52.5	0	0	0	2	1	0	0	0	0	0	0	4	0	0	0
57.5	1	1	2	1	2	0	4	0	0	2	0	12	0	4	0
62.5	8	0	1	0	2	0	1	0	0	4	0	11	0	0	0
67.5	2	0	1	2	7	0	5	1	0	1	0	14	3	2	0
72.5	1	0	2	0	5	0	1	0	1	1	0	16	0	10	0
77.5	2	1	6	0	5	0	2	0	1	0	0	26	5	10	1
82.5	1	1	0	1	4	0	7	1	0	0	2	7	5	16	0
87.5	3	1	0	0	1	0	14	0	0	0	0	1	5	9	0
92.5	0	3	1	1	4	0	1	1	0	0	1	2	2	13	0
97.5	0	1	2	0	0	1	6	0	0	0	0	3	2	7	0
102.5	0	0	1	0	1	0	3	0	0	0	0	0	3	3	0
107.5	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0
112.5	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0
117.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(c) cod predation on cod

Predator length (cm)	Year			
	1985	1987	1991	1994
2.5	0	0	0	0
7.5	0	0	0	0
12.5	0	0	0	0
17.5	0	0	0	0
22.5	0	0	0	0
27.5	0	0	0	0
32.5	0	0	0	0
37.5	0	0	0	0
42.5	0	0	0	1
47.5	0	1	0	0
52.5	0	2	0	0
57.5	0	0	0	0
62.5	0	0	1	0
67.5	0	0	0	0
72.5	0	0	0	0
77.5	0	0	1	0
82.5	0	0	0	0
87.5	1	0	0	0
92.5	0	0	0	0
97.5	0	0	0	0
102.5	0	0	0	0
107.5	0	0	0	0
112.5	0	0	0	0
117.5	0	0	0	0

Table A.23. Breakdown of the diet of each prey species by cod of various lengths (N/A denotes cases in which the prey length exceeds the predator length)

(a) cod predation on pollock

Predator length (cm)	Prey length (cm)													
	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5
42.5	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	N/A	N/A	N/A	N/A	N/A	N/A
47.5	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	N/A	N/A	N/A	N/A	N/A
57.5	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	N/A	N/A	N/A
62.5	0.000	0.500	0.200	0.100	0.100	0.000	0.100	0.000	0.000	0.000	0.000	0.000	N/A	N/A
67.5	0.000	0.063	0.000	0.000	0.000	0.188	0.313	0.375	0.000	0.063	0.000	0.000	0.000	N/A
72.5	0.000	0.077	0.154	0.000	0.077	0.077	0.308	0.154	0.077	0.077	0.000	0.000	0.000	0.000
77.5	0.071	0.000	0.000	0.000	0.000	0.071	0.000	0.214	0.571	0.000	0.071	0.000	0.000	0.000
82.5	0.000	0.000	0.000	0.000	0.091	0.091	0.091	0.091	0.364	0.273	0.000	0.000	0.000	0.000
87.5	0.000	0.000	0.000	0.000	0.000	0.071	0.214	0.286	0.357	0.071	0.000	0.000	0.000	0.000
92.5	0.100	0.000	0.000	0.000	0.000	0.000	0.200	0.100	0.500	0.100	0.000	0.000	0.000	0.000
97.5	0.000	0.000	0.000	0.000	0.000	0.000	0.143	0.143	0.000	0.429	0.286	0.000	0.000	0.000
107.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000
112.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000

(b) cod predation on mackerel

Predator length (cm)	Prey length (cm)									
	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5
42.5	0.333	0.000	0.333	0.000	0.333	0.000	0.000	0.000	N/A	N/A
47.5	0.000	0.000	0.250	0.500	0.250	0.000	0.000	0.000	0.000	N/A
52.5	0.000	0.000	0.143	0.429	0.286	0.143	0.000	0.000	0.000	0.000
57.5	0.065	0.032	0.097	0.548	0.194	0.065	0.000	0.000	0.000	0.000
62.5	0.030	0.000	0.091	0.303	0.364	0.152	0.030	0.030	0.000	0.000
67.5	0.023	0.023	0.114	0.477	0.159	0.091	0.046	0.068	0.000	0.000
72.5	0.021	0.000	0.063	0.271	0.292	0.250	0.083	0.021	0.000	0.000
77.5	0.095	0.000	0.159	0.302	0.175	0.175	0.048	0.016	0.016	0.016
82.5	0.000	0.000	0.019	0.148	0.296	0.296	0.185	0.037	0.019	0.000
87.5	0.000	0.000	0.054	0.216	0.297	0.243	0.189	0.000	0.000	0.000
92.5	0.138	0.000	0.103	0.069	0.103	0.379	0.103	0.103	0.000	0.000
97.5	0.130	0.000	0.044	0.087	0.087	0.261	0.217	0.174	0.000	0.000
102.5	0.000	0.000	0.000	0.063	0.188	0.500	0.188	0.063	0.000	0.000
107.5	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
112.5	0.000	0.000	0.000	0.000	0.500	0.000	0.500	0.000	0.000	0.000

(c) cod predation on cod

Predator length (cm)	Prey length (cm)																	
	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5	82.5	87.5+
42.5	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
47.5	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
52.5	0.500	0.500	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
62.5	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	N/A	N/A	N/A	N/A	N/A	N/A
77.5	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	N/A	N/A	N/A
87.5	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	N/A

Table A.24. The fishery weights-at-age (kg) for pollock.

	Age (years)															
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1979	0	0	0.231	0.348	0.529	0.731	0.673	0.825	0.944	0.953	1.038	1.164	1.060	1.519	1.579	1.021
1980	0	0	0.239	0.553	0.765	0.841	0.863	0.913	1.000	1.089	1.063	1.020	1.157	1.102	0.852	1.524
1981	0	0	0.000	0.478	0.552	0.729	0.764	0.782	0.810	0.895	0.902	0.860	1.020	1.026	0.893	0.908
1982	0	0	0.000	0.418	0.541	0.644	0.784	0.822	0.842	0.892	0.984	1.001	0.958	0.955	0.906	0.966
1983	0	0	0.000	0.418	0.541	0.733	0.780	0.795	0.926	0.957	1.015	0.902	0.958	0.955	0.974	0.966
1984	0	0	0.426	0.446	0.671	0.742	0.810	0.872	0.968	0.996	1.270	1.643	1.135	1.221	0.974	0.966
1985	0	0	0.467	0.566	0.671	0.690	0.803	0.854	0.857	1.091	1.233	1.600	1.664	1.221	1.645	1.093
1986	0	0	0.000	0.511	0.602	0.747	0.827	0.870	0.951	0.927	1.014	0.943	1.070	0.896	1.645	1.093
1987	0	0	0.000	0.511	0.685	0.756	0.833	0.850	0.872	0.981	1.072	0.992	1.338	1.155	1.006	1.093
1988	0	0	0.000	0.474	0.685	0.801	0.790	0.821	0.928	0.888	0.984	0.893	0.784	0.722	0.898	1.062
1989	0	0	0.339	0.474	0.661	0.754	0.851	0.926	0.993	1.061	1.111	1.150	1.189	1.150	1.194	1.140
1990	0	0	0.000	0.478	0.552	0.729	0.764	0.782	0.810	0.895	0.902	0.860	1.020	1.026	0.893	0.908
1991	0	0	0.000	0.478	0.667	0.655	0.799	0.962	1.076	1.173	1.099	1.218	1.157	1.096	1.290	1.086
1992	0	0	0.000	0.478	0.640	0.742	0.725	0.797	0.936	1.246	1.027	1.003	1.250	1.145	1.051	1.098
1993	0	0	0.000	0.551	0.886	0.824	1.033	1.032	1.140	1.081	1.164	1.191	1.203	1.326	1.137	1.135
1994	0	0	0.000	0.551	0.637	0.844	0.974	1.136	1.140	1.122	1.191	1.244	1.266	1.059	1.090	1.152
1995	0	0	0.000	0.551	0.847	0.844	1.126	1.330	1.397	1.355	1.433	1.420	1.501	1.447	1.658	1.321
1996	0	0	0.000	0.539	0.475	0.930	1.029	1.180	1.275	1.395	1.468	1.355	1.378	1.362	1.456	1.301
1997	0	0	0.000	0.471	0.619	0.835	1.006	1.119	1.180	1.324	1.391	1.419	1.328	1.332	1.408	1.377
1998	0	0	0.000	0.403	0.763	0.740	0.983	1.058	1.085	1.253	1.314	1.483	1.278	1.301	1.360	1.452
1999	0	0	0.000	0.403	0.763	0.740	0.983	1.058	1.085	1.253	1.314	1.483	1.278	1.301	1.360	1.452
2000	0	0	0.000	0.403	0.763	0.740	0.983	1.058	1.085	1.253	1.314	1.483	1.278	1.301	1.360	1.452
2001	0	0	0.000	0.403	0.763	0.740	0.983	1.058	1.085	1.253	1.314	1.483	1.278	1.301	1.360	1.452
2002	0	0	0.000	0.403	0.763	0.740	0.983	1.058	1.085	1.253	1.314	1.483	1.278	1.301	1.360	1.452
2003	0	0	0.000	0.403	0.763	0.740	0.983	1.058	1.085	1.253	1.314	1.483	1.278	1.301	1.360	1.452

Table A.25. The fishery weights-at-age for mackerel.

	Age (years)															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
wt (kg)	0	0.1	0.185	0.398	0.549	0.656	0.732	0.785	0.823	0.850	0.869	0.882	0.892	0.899	0.903	0.907

Table A.26. The fishery weights-at-age for cod.

	Age (years)												
	0	1	2	3	4	5	6	7	8	9	10	11	12
wt (kg)	0	0.1	0.3	0.6	1.5	2.6	3.5	5.1	6.8	7.5	8.2	8.4	9.9

Table A.27. The survey weights-at-age (kg) for pollock.

	Age (years)															
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1979	0.000	0.137	0.382	0.655	0.901	1.101	1.254	1.367	1.449	1.506	1.547	1.575	1.595	1.609	1.618	1.625
1980	0.000	0.137	0.382	0.655	0.901	1.101	1.254	1.367	1.449	1.506	1.547	1.575	1.595	1.609	1.618	1.625
1981	0.000	0.098	0.302	0.536	0.747	0.917	1.046	1.140	1.208	1.255	1.287	1.310	1.325	1.336	1.343	1.348
1982	0.000	0.098	0.302	0.536	0.747	0.917	1.046	1.140	1.208	1.255	1.287	1.310	1.325	1.336	1.343	1.348
1983	0.000	0.059	0.223	0.417	0.593	0.733	0.838	0.914	0.966	1.003	1.028	1.044	1.056	1.063	1.068	1.072
1984	0.000	0.079	0.236	0.416	0.582	0.717	0.822	0.899	0.955	0.995	1.024	1.044	1.058	1.068	1.074	1.079
1985	0.000	0.079	0.236	0.416	0.582	0.717	0.822	0.899	0.955	0.995	1.024	1.044	1.058	1.068	1.074	1.079
1986	0.000	0.098	0.249	0.416	0.571	0.701	0.805	0.884	0.944	0.988	1.020	1.043	1.060	1.072	1.081	1.087
1987	0.000	0.098	0.254	0.452	0.686	0.804	0.944	1.036	1.084	1.123	1.191	1.195	1.179	1.190	1.208	1.173
1988	0.000	0.098	0.254	0.452	0.686	0.804	0.944	1.036	1.084	1.123	1.191	1.195	1.179	1.190	1.208	1.173
1989	0.000	0.098	0.254	0.452	0.686	0.804	0.944	1.036	1.084	1.123	1.191	1.195	1.179	1.190	1.208	1.173
1990	0.000	0.098	0.254	0.452	0.686	0.804	0.944	1.036	1.084	1.123	1.191	1.195	1.179	1.190	1.208	1.173
1991	0.000	0.000	0.259	0.488	0.801	0.906	1.083	1.188	1.224	1.257	1.363	1.348	1.297	1.309	1.336	1.260
1992	0.000	0.074	0.256	0.439	0.778	0.898	1.007	1.129	1.201	1.235	1.340	1.353	1.343	1.354	1.374	1.356
1993	0.000	0.074	0.256	0.439	0.778	0.898	1.007	1.129	1.201	1.235	1.340	1.353	1.343	1.354	1.374	1.356
1994	0.000	0.074	0.256	0.439	0.778	0.898	1.007	1.129	1.201	1.235	1.340	1.353	1.343	1.354	1.374	1.356
1995	0.000	0.074	0.256	0.439	0.778	0.898	1.007	1.129	1.201	1.235	1.340	1.353	1.343	1.354	1.374	1.356
1996	0.000	0.074	0.256	0.439	0.778	0.898	1.007	1.129	1.201	1.235	1.340	1.353	1.343	1.354	1.374	1.356
1997	0.000	0.070	0.253	0.390	0.756	0.889	0.931	1.070	1.179	1.213	1.316	1.358	1.389	1.399	1.413	1.452
1998	0.000	0.051	0.234	0.419	0.720	0.990	1.038	1.190	1.242	1.377	1.453	1.485	1.537	1.539	1.543	1.530
1999	0.000	0.051	0.234	0.419	0.720	0.990	1.038	1.190	1.242	1.377	1.453	1.485	1.537	1.539	1.543	1.530
2000	0.000	0.051	0.234	0.419	0.720	0.990	1.038	1.190	1.242	1.377	1.453	1.485	1.537	1.539	1.543	1.530
2001	0.000	0.051	0.234	0.419	0.720	0.990	1.038	1.190	1.242	1.377	1.453	1.485	1.537	1.539	1.543	1.530
2002	0.000	0.033	0.215	0.449	0.683	1.091	1.145	1.309	1.306	1.542	1.591	1.611	1.686	1.680	1.672	1.608
2003	0.000	0.033	0.215	0.449	0.683	1.091	1.145	1.309	1.306	1.542	1.591	1.611	1.686	1.680	1.672	1.608

Table A.28. The survey weights-at-age for mackerel.

	Age (years)															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
wt (kg)	0.000	0.100	0.128	0.421	0.660	0.756	0.794	0.810	0.816	0.818	0.819	0.820	0.820	0.820	0.820	0.820

Table A.29. Population weights and maturity-at-age. The months in which spawning is assumed to occur are March, August, and February for pollock, mackerel and cod respectively.

	Age (years)															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Weight-at-age																
Pollock	0.000	0.040	0.210	0.410	0.690	0.890	1.090	1.250	1.330	1.400	1.480	1.510	1.640	1.540	1.510	1.590
Mackerel	0.000	0.100	0.128	0.421	0.660	0.756	0.794	0.810	0.816	0.818	0.819	0.820	0.820	0.820	0.820	0.820
Cod	0.000	0.100	0.300	0.600	1.500	2.600	3.500	5.100	6.800	7.500	8.200	8.400	9.900			
Maturity-at-age																
Pollock	0.000	0.000	0.008	0.289	0.641	0.842	0.901	0.947	0.963	0.970	0.978	0.984	0.990	1.000	1.000	1.000
Mackerel	0.000	0.037	0.224	0.688	0.944	0.992	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Cod	0.000	0.001	0.005	0.019	0.090	0.300	0.514	0.750	0.880	0.920	0.940	0.950	0.970			

Table A.30. Age-length transition matrix for pollock.

	Length (cm)													
Age	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5
0	0.0023	0.9953	0.0023	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.0002	0.1402	0.7772	0.0823	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0007	0.0522	0.4368	0.4516	0.0579	0.0008	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0000	0.0005	0.0137	0.1177	0.3506	0.3659	0.1338	0.0170	0.0007	0.0000	0.0000
4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0032	0.0683	0.3477	0.4355	0.1350	0.0101	0.0002	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0010	0.0208	0.1485	0.3743	0.3359	0.1071	0.0120	0.0005
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0014	0.0188	0.1115	0.2938	0.3456	0.1816	0.0425	0.0044
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0018	0.0279	0.1632	0.3669	0.3189	0.1070	0.0137
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0102	0.1141	0.3776	0.3755	0.1122	0.0099
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0073	0.0672	0.2499	0.3777	0.2330	0.0584
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0010	0.0245	0.1821	0.4188	0.3014	0.0675
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0053	0.0419	0.1639	0.3150	0.2984	0.1393
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0086	0.1022	0.3614	0.3881	0.1267
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0025	0.0245	0.1178	0.2789	0.3260	0.1883
14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0105	0.1121	0.3670	0.3766	0.1211
15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0027	0.0250	0.1155	0.2704	0.3217	0.1946

Table A.31. Age-length transition matrix for mackerel.

	Length (cm)									
Age	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5
0	0.0296	0.9408	0.0296	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.0006	0.0198	0.1800	0.4419	0.2994	0.0555	0.0027	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0164	0.5035	0.4673	0.0128	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0000	0.0033	0.1587	0.6095	0.2218	0.0067	0.0000
4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0021	0.1815	0.6734	0.1418	0.0012
5	0.0000	0.0000	0.0000	0.0000	0.0002	0.0082	0.1083	0.3862	0.3832	0.1058
6	0.0000	0.0000	0.0000	0.0000	0.0001	0.0048	0.0729	0.3221	0.4194	0.1619
7	0.0000	0.0000	0.0000	0.0000	0.0001	0.0030	0.0515	0.2676	0.4292	0.2142
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0027	0.0475	0.2556	0.4288	0.2262
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0027	0.0475	0.2556	0.4288	0.2262
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0027	0.0475	0.2556	0.4288	0.2262
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0027	0.0475	0.2556	0.4288	0.2262
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0027	0.0475	0.2556	0.4288	0.2262
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0027	0.0475	0.2556	0.4288	0.2262
14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0027	0.0475	0.2556	0.4288	0.2262
15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0027	0.0475	0.2556	0.4288	0.2262

Table A.32. Age-length transition matrix for cod.

	Length (cm)																							
Age	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5	82.5	87.5	92.5	97.5	102.5	107.5	112.5	117.5
0	0.0352	0.9296	0.0352	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.0050	0.1215	0.4859	0.3443	0.0424	0.0008	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0002	0.0040	0.0406	0.1805	0.3496	0.2964	0.1098	0.0177	0.0012	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0003	0.0079	0.0726	0.2638	0.3812	0.2195	0.0501	0.0045	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0039	0.0473	0.2189	0.3882	0.2650	0.0694	0.0069	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0021	0.0457	0.2692	0.4455	0.2091	0.0274	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0034	0.0849	0.4117	0.4117	0.0849	0.0034	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0006	0.0222	0.2039	0.4648	0.2685	0.0387	0.0013	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0045	0.0430	0.1816	0.3440	0.2933	0.1124	0.0193	0.0015	0.0000	0.0000	0.0000	0.0000
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0013	0.0093	0.0416	0.1196	0.2214	0.2639	0.2028	0.1004	0.0320	0.0066	0.0009	0.0001	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0005	0.0074	0.0525	0.1848	0.3232	0.2811	0.1215	0.0260	0.0028	0.0001	0.0000	0.0000
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0020	0.0130	0.0533	0.1402	0.2375	0.2593	0.1825	0.0827	0.0241	0.0045	0.0005	0.0000
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0008	0.0059	0.0277	0.0867	0.1805	0.2500	0.2304	0.1414	0.0577	0.0157	0.0028	0.0004